GEOMETRICAL RECONSTRUCTION FROM SINGLE LINE DRAWINGS USING OPTIMIZATION-BASED APPROACHES

Company, P.; Gomis, J.M. [§] and Contero, M.[1]

Universitat Jaume I. (Dept. de Tecnología)
Campus del Riu Sec. E12071 Castellón. Spain
pcompany@tec.uji.es, contero@tec.uji.es

[§] Universidad Politécnica de Valencia. (Dept. de Expresión Gráfica en la Ingeniería)
Camino de Vera S/N. E46022 Valencia. Spain
jmgomis@degi.upv.es

ABSTRACT

Optimization is one of the most promising geometrical reconstruction approaches. In this approach, the 2D vertices of the given figure maintain their plane coordinates (X,Y), while a set of Z coordinates (orthogonal to the plane) is computed to obtain a 3D configuration that matches the "implicit spatial information" contained in the departure drawing. In other words, Z coordinates are the variables, and image regularities are used to define both the Objective Function and the Constraints. Some authors have introduced and tested the approach. Nevertheless, further improvements are needed. Mainly because in this problem only global optimum is acceptable in order to ensure the "psychologically plausible" model is always the one to be obtained. In this paper, some key aspects of the strategy proposed by the authors to convey the optimization process towards the psychologically plausible solution are discussed.

Keywords: geometrical reconstruction, optimization.

INTRODUCTION

Geometrical Reconstruction is the name usually given to the problem of how to recover the geometrical information of 3D objects contained on their 2D images. The origins of geometrical reconstruction can be traced with a reduced set of references [Nagen88], [Wang93], [Gomis96]. And the present situation can be summarized pointing out that most of the known reconstruction approaches are now in experimental stages, and can interpret (with few errors), all kind of polyhedral objects. Interpretation of the most usual 3D primitives (like cylinders, spheres, etc.) is

[1] This work was supported by the authors' regional government ("Generalitat Valenciana"), under its granting program for Technological Research and Development (Project GV97-TI-04-39, titled "3D Object Reconstruction from 2D Representations").

also considered by some of the approaches. Anyway, when object complexity increases, automatic processes usually give pass to semiautomatic ones.

Geometrical Reconstruction may be considered as a small part of the more complex field of "Artificial Vision" or "Computer Vision". Understood as the field where all those tools and strategies employed to make the computers to "see" (that is, to make sense on all the images they can get) is developed. In fact, Artificial Vision can be divided into two parts; where the first one is more concerned about 3D scenes, and the second one deals with 2D images.

When 2D images constitute the input we can, again, divide the problem in two different aspects. First, the one related to some phenomenon we want to study and concerned on how to make sense from images obtained through all kind of cameras, scanners and sensors. Here, the objective is filtering the obtained images to describe them in a compact or convenient form, or to highlight some pieces of information presents on it. Second, we can find some researchers trying to add the capabilities the computers need to "read" information in the way humans can read it. Character recovery, interpretation of symbols and signs, and, finally, making computer able to interpret 2D images in terms of 3D model projections. This problem is named "reconstruction" (or more precisely "geometrical reconstruction") and implies determination of geometrical and topological relations of all atomic parts of one object.

In this context, it must be pointed out that, from the point of view of Geometry, it has been always very well known that full recovery of a geometrical 3D model from one single projection of it is not possible. Nevertheless, from the field of psychologists it is also very well known the fact that humans seem to have no problems to identify 3D models depicted in 2D images. What is more, it seems to have a great consensus about which is the "correct" and "single" model all humans see in every picture. The reason comes from the fact that humans, when "read" drawings, do implicit recovery actions. According to Gestalt school, this is because human perception holds some common characteristics, called "principles of organization".

Moreover, implicit assumptions are made in a tentative and iterative way. At first glance, humans make some assumptions on which 3D model is represented in a picture. Latter this initial assumption is validated or modified. Depending on when the supposed model does exactly match not only the given image, but also the previous experience and intuition the viewer has about what kind of objects can correspond to the given image and which was the intention of the author.

Consequently, an iterative process where some initial solution is refined according with some perceived characteristics, seems to be a good strategy to get what the sole application of geometry rules cannot obtain: a 3D psychologically plausible model. This process considers the same assumptions a human can made, and discards those initial assumptions they are proved to be no valid.

OPTIMIZATION APPROACH

The Reconstruction problem can be described in terms of a mathematical optimization problem, departing from the key idea, proposed by Marill [Maril91], that (X,Y) coordinates of every point in the graph are equal to (X,Y) coordinates of corresponding vertices in the model. Thus, the problem can be described as:

Minimize $F(\mathbf{z})$,

Where:

- $\mathbf{z} = (z_1, z_2,...,z_n)$ is the set of z-coordinates that constitute the independent variables vector.
- n is the number of vertices or "control points" that fully determine the shape of the model, and determine the "order" or the problem.

In other words, Z coordinates are the variables. As a consequence, an infinite number of models are "valid", because they all can be projected in the same figure. Marill called "orthographic extension" to the full set of infinite three-dimensional objects whose orthogonal projection is equal to the given graph. Some of the model belonging to the orthographic extension of a given 2D graph are shown in figure 1.

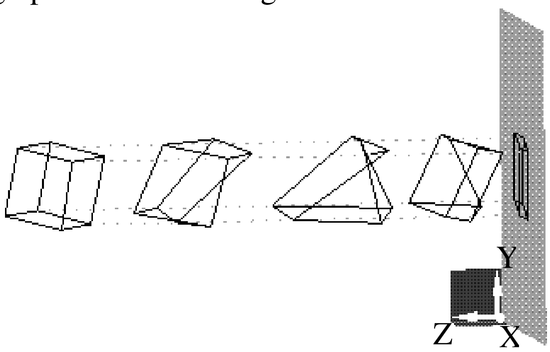

Figure 1

The second key idea in the proposed approach is to develop some criteria to choose one particular model among the infinite ones contained in the Extension. As was said before, there is, in general, only one "good" solution, in terms of "psychological plausibility". That is, human observers seem to have no doubts to choose the appropriate solution among all those constituting the Orthographic Extension, as Leclerc and Fischler perceived [Lecle92].

"Regularities" are the way to make explicit such human perceptions. The foundations of this concept can be traced back in classic reference books like the one by Koffka [Koffk67]. Nevertheless, the concept of regularity requires a more detailed explanation. In fact the first appearances of the word "regularities" associated with graphical representations are due to the Gestalt Psychologists, who named regularities to "those relations that cannot be an accident". Latter on, when some Scientific Visualization areas, related with semantic perception, began their own research, the term "regularities" became synonymous of "template to describe images in a compact or convenient form". Finally, in the geometrical reconstruction field, regularities are interpreted as "those properties of the image that must correspond to some properties the searched model also has" (See [Lipso95], [Sphit96] and [Lipso96]).

Consequently, it is supposed that, by inspecting the image, the necessary number of properties describing how the model must be can be discovered. Then, enough of the conditions the good solution must fit can be formulated in a mathematical language. Hence, the Objective Function can be easily defined in terms or regularities in the following form:

$$F = \Sigma\, \alpha_j\, R_j(\mathbf{z}) \qquad (1)$$

where,

- α_j is the j-th weighting coefficient, and
- $R_j(\mathbf{z})$ is the j-th regularity.

Regularities must be expressed in terms of the independent variables ($\mathbf{z}$), and must be formulated to be equal to zero when a complete compliance of the condition is achieved, and very different from zero for clear non-compliance.

OPTIMIZATION PROBLEM

Reconstruction differs from other optimization problems in a capital fact: local minimums are undesired solutions, because only the 3D model that user would have constructed by himself will be accepted. In other words, it never minds how close the objective functions of a local minimum can be to the "real" solution, because the desired solution must fit psychological desires, not numerical differences. Hence, while a local minimum, or simply a significant improvement in the Objective Function, may be a good result in other fields, a similar reduction in the Objective Function of a Geometrical Reconstruction problem do not solve any problem. Because the local minimum will correspond to one component of the orthographic extension whose shape can differ significantly from the psychologically plausible solution.

In a structural optimization problem, where the Objective Function measures the cost of the structure, any reduction on its value is a good result, because it leads to a cheaper and safer design than the initial one. But this is not the case in Geometrical Reconstruction. A local minimum in Geometrical Reconstruction means a model that significantly differs from the one the user wanted to obtain. Consequently, the user will not accept the reconstruction process as a valid and comfortable mean to communicate with the computer. So, introducing those numerical optimization algorithms that claim to be able to find global minimum (see [Press88]) seems to be the obvious solution. "Simulated Annealing" (SA) is one of such promising approaches because it provides a method to obtain good solutions to difficult optimization problems.

The application of SA in different fields began with the work by Kirkpatrick et al. [Kirkp83]. They showed how a mathematical model to simulate the behavior of molecules in a liquid that freezes and crystallize, or atoms in a metal that cools and anneals (as proposed by Metropolis et al. [Metro53]), could be used for global optimization purposes.

In the optimization analogy, the objective function to be minimized is the equivalent of the solid energy, and the molecules are replaced by the optimization variables. To simulate the cooling process, a generation mechanism is applied to get a neighbor Objective Function. To simulate the effect of annealing on the thermal state of solid, one parameter, called "temperature", controls the acceptance of such tentative neighbor objective functions. In other words, the "moves" (the variations in the variables) are selected to get a new objective function. But, whilst other algorithms never accept moves towards a worst objective function, SA algorithms *sometimes* accept such kind of moves. This allows the algorithm to escape from local minimum because in some cases a bad move can be accepted. In fact, the Metropolis acceptance criterion has a high probability to accept a bad move at the beginning of the cooling process, when the temperature is high. As the temperature decreases, it becomes more and more difficult to accept such kind of transitions.

Following is reproduced in pseudocode the structure of a SA algorithm, to allow a deeper understanding on it:

select an initial configuration $i \in \mathbf{z}$;
select an initial temperature $T > 0$;
set temperature change counter $c = 0$;
  repeat
    set repetition counter $n = 0$;
    repeat
      generate neighbor configuration $k \in \mathbf{z}$;
      evaluate Objective Function variation $\Delta F = F(k) - F(i)$
      if $\Delta F < 0$ **then** $i := k$
      else if rand(0,1)<exp($-\Delta F/T$)**then** $i := k$
      $n := n+1$
    until equilibrium criterion true;
    set temperature counter $c := c + 1$;
    select new temperature $T := T(c)$;
until stopping criteria true

The parameters controlling the SA can be classified in the two types shown in table 1.

Table 1

Generic:	Problem specific:
Initial temperature	Configuration space
Cooling schedule	Neighborhood structure
Equilibrium criterion	Cost function
Stopping criterion	Initial configuration

OUR RECONSTRUCTION ALGORITHM

A program called REFER has been developed by the authors [Compa98]. The application was developed using Microsoft's Developer Studio, to run on PC platforms under Windows NT or Windows 95. C++ was used to implement the calculations and data management. Graphical User Interaction was achieved by calling win32 operations through Visual C++. And Open GL was chosen as the best alternative to generate outputs of 3D models.

In the interface, special interest has the window where optimization process is monitored. This window (see figure 2), displays the Objective Function evolution with some additional parameters (like the "temperature"). Moreover, by double clicking in the window, an alternative graph showing variables evolution appears.

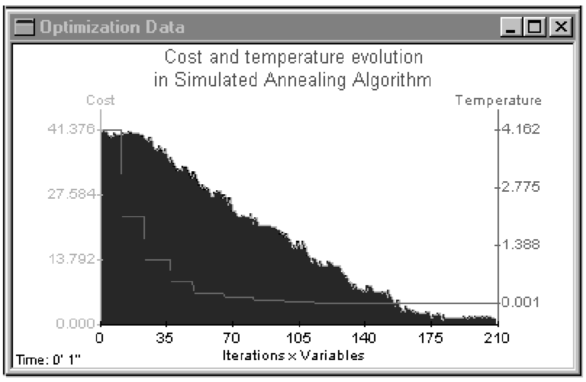

Figure 2

OPTIMIZATION STRATEGY

In order to use the SA algorithm to solve reconstruction problems, the *problem specific parameters* shown in table 1, must be adapted. In this sense, we have considered the following ones:

- Configuration space: defined by **z,** the set of z-coordinates.
- Neighborhood structure: to generate a new configuration. The generation is made by simply applying a certain displacement to one of the z_i coordinates randomly chosen.
- Cost function. Following the same strategy explained in previous sections, the cost function is based on regularities.
- Initial configuration. In spite of SA being a global optimization strategy, the initial configuration still is important to achieve a good performance. The strategies vary from making all Z coordinates equal to 0, to defining a deterministic inflation process to obtain a "tentative" 3D model.

The configuration space for the optimization problems, depends on the nature the objects to be reconstructed have. When polyhedral objects are considered, the points whose z-coordinates are calculated are those junctions in the departure graph that are candidate to correspond to 3D vertices in the model. In a more general context, control points of some curves in the departure graph must be used as candidate to correspond to control points in some 3D curves or surfaces in the model.

To select the displacement value, two strategies have been tested: a) to use a fixed quantity, or b) to use a variable value. Best results are obtained with variable values. Especially when the variation is a smooth decrement. In this way, greater variations are allowed in the initial steps, and fine adjusts are obtained in the final part of the process. In both cases, the range of the displacement must be proportional to the 2D coordinate range. That is, the range in Z must be proportional to the range in both x and y [Compa98]. In particular, we use the minimum between x_{max}-x_{min} and y_{max}-y_{min}, where (x,y) are the coordinates of all vertices in the departure graph.

As was said in the previous section, the Objective Function, must be formulated in terms of regularities, and in a way to ensure it to be equal to zero when a complete compliance of all regularities is achieved. Nevertheless, this is a critical aspect in the problem configuration. So we will explain it in more detail in the next section.

Relevant to the selection of one initial solution, it must be pointed out that the common strategy up to now is making all Z coordinates equal to 0 (i.e. the departure 2D graph is used as initial solution). Unfortunately, the graph is a local minimum, because many regularities are trivially accomplished on it. For instance, a loop made by some edges (and, supposed to correspond to a plane face in the model), always determine a trivial plane face in the graph.

In the "inflation" strategy, proposed by Leclerc and Fischler, the Objective Function was balanced with a varying parameter λ in the following form: $F= F_A + \lambda F_I + (1-\lambda) F_O$. Where F_I (the "inflation" part) includes a set of conditions that are not trivially accomplished in the initial solution ($\mathbf{z}$=0), but are not true regularities, and, hence, do not need to be accomplished in the optimum. On the contrary, F_O (the "optimization" part) contains those true regularities that are trivially accomplished in the initial solution. Finally, F_A includes all those regularities that can be always applied, because they are not trivially accomplished in the initial solution.

The strategy was defined to allow the optimization algorithm to escape from trivial solution using inflation criteria, while guarantying that the final solution depends only on true regularities. Nevertheless, undesired oscillations can happen, and some intermediate local minimum can be the result of the inflation part. Consequently, alternative initial solutions strategies must be developed, to avoid the use of the "inflation" strategy.

The best alternative seems to be to select as initial solution someone that accomplishes some rules. In other words, the approach is to define a deterministic inflation process to obtain a "tentative" 3D model whose topology is as close as possible to the final one. This is the case of "preliminary reconstruction" proposed by Lipson and Sphitalni [Lipso96], and based on analyzing the angular distribution graph of lines, to obtain the prevailing angles, and hence the main directions. When three prevailing angles are obtained, the orthogonal perceptual rule ("those angles must correspond to three orthogonal directions") is applied to obtain a tentative model.

This is clearly one area where more research is needed to define better deterministic rules to obtain tentative models.

In the other hand, *generic parameters* control the behavior of the algorithm itself, and must be tuned to improve the performance. But they are supposed not to have a capital effect on the final result. Our own experience makes us believe this is not always completely true. In fact we already detected some significant deviations from global optimum, due to different tuning of those "generic" parameter. Anyway, the initial values and criteria chosen were those commonly used in other simulated annealing implementations. Those parameters can be grouped in four main aspects described in table 1. In our own implementation, we have reproduced this scheme (see the dialog box reproduced in figure 3):

- Initial temperature: using the methodology proposed by Kouvelis et al. [Kouve92] a minimum acceptance ratio is fixed to determine the initial temperature.
- Cooling schedule: we use the classic proportional evolution law.
- Equilibrium criterion: we fix a maximum number of iterations for each temperature step related to the dimension of the configuration space
- Stopping criterion: The program stops when the present solution cannot be significantly improved, or when a maximum of attempts has been done.

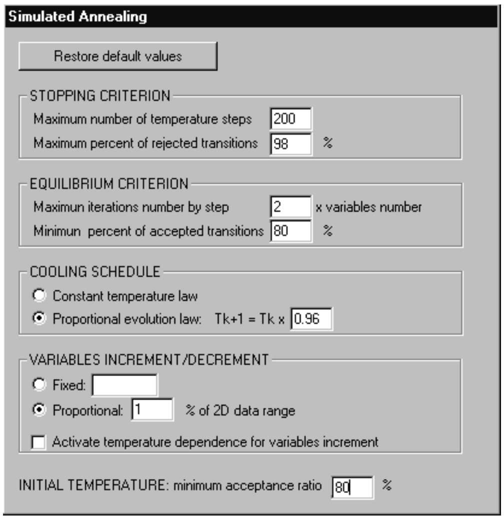

Figure 3

REGULARITIES AND CONSTRAINTS

When regularities are introduced in the Objective Function, what is done is including penalties for the case where some regularities are not matched in the solution. Tat means that full accomplishment is not strictly enforced. Hence, models obtained through this method have slight "faults". For instance, when a cube is reconstructed, the resulting angles are "very close" to 90º, but they can deviate from the "exact" solution the user desired.

Our proposal, in a next step, to solve this limitation is to convert some regularities imposed in one optimization process into constraints for the next one. This means linking successive but independent optimization processes. Every optimization process uses the final solution achieved in the previous one as initial solution. But the objective function changes. Because those regularities that are considered to be true are converted into constraints, while the others remain in the objective function.

In the other hand, the weighting coefficients (α_j) included in the Objective Function (Eq. 1) have a capital importance. Because, depending on the formulation adopted to represent every regularity in terms of the variables ($R_j(\mathbf{z})$), the range of values may vary a lot from one regularity to other. Thus, some normalization is needed to prevent the algorithm to be more sensitive to some regularities.

To sum up, optimization approaches still need a better definition of regularities. Even more, may be some kind of hierarchical structure of regularities would be defined to determine their priorities and weights when assembling the objective function. It must be noticed that redundancies and incorrect balancing of regularities may lead not only to inefficiency, but can force not plausible solutions to be selected as global optimum.

CONCLUSIONS

Geometrical Reconstruction is one of those fields concerned on adding "reading capabilities" to computers. In particular, it is concerned in recovering the 3D geometrical model represented in any 2D geometrical image. In this context, Optimization approach is becoming a good alternative. Nevertheless, geometrical reconstruction has one key difference related to other field where optimization is been successfully used. Only the psychologically plausible solution is acceptable in the reconstruction problem. That means no one good solution is valid, only the best one is acceptable.

Hence, global optimization algorithms are needed. Simulated Annealing methods are seen as good alternatives in this context. Nevertheless, for the algorithm to find the correct solution, a non-trivial condition must be accomplished: the Objective Function minimum must fit the psychologically plausible solution. And, if constraints are defined, this global minimum must also be in the feasible region. Thus, to ensure all those conditions can take place, a consistent definition of regularities is needed.

In a nutshell: the success of optimization approaches requires a correct tuning of a *global* optimization strategy, and a very careful choice and weighting of all the regularities that both, the Objective Function and the Constraints, must contain.

REFERENCES

Company P., Gomis J.M., and Contero M., 1998. An Optimization-Based Algorithm to Reconstruct 3D Models from Single Line Drawings. *II Seminario Italo-Spagnolo. Progettazione e Fattibilità dei Prodotti Industriali*, (Not yet published).

Gomis J.M. and Company P., 1996. Reconstrucción geométrica tridimensional. *Anales de Ingeniería Gráfica*, No. 2, pp 26-34.

Kirkpatrick S., Gelatt C.D. and Vecchi M.P., 1983. Optimization by simulated annealing. *Science*, vol. 220, No. 4598, pp. 671-680.

Koffka K., 1935 (1967). *Principles of Gestalt Psychology.* Harcourt Brace, New York.

Kouvelis P., Chiang W. and Fitzsimmons J., 1992. Simulated annealing for machine layout problems in the presence of zoning constraints. *European Journal of Operational Research*, vol. 57, pp. 203-223.

Leclerc, Y. and Fischler M., 1992. An Optimization-Based Approach to the Interpretation of Single Line Drawings as 3D Wire Frames. *International Journal of Computer Vision.* Vol. 9, No. 2, pp. 113-136.

Lipson H. and Shpitalni M., 1995. A New Interface of Conceptual Design Based on Object Reconstruction From a Single Freehand Sketch. *Ann CIRP.* Vol. 44, No. 1, pp. 133-136.

Lipson H. and Shpitalni M., 1996. Optimization-Based Reconstruction of a 3D Object from a Single Freehand Line Drawing. *Computer Aided Design.* Vol. 28, No. 8, pp. 651-663.

Marill, T., 1991. Emulating the Human Interpretation of Line-Drawings as Three-Dimensional Objects. *International Journal of Computer Vision.* Vol. 6, No. 2, pp. 147-161.

Metropolis N., Rosenbluth A.W., Rosenbluth M.N. and Teller A.H., 1953. Equation of state calculations by fast computing machines. *The Journal of Chemical Physics*, vol. 21, no. 6, pp. 1087-1091.

Nagendra I.V. and Gujar U.G., 1988. 3-D Objects From 2-D Orthographic Views – A Survey. *Computers & Graphics*. Vol. 12, No. 1, pp. 111-114.

Press, W.H.; Flannery, B.P.; Teukolsky, S.A. and Vetterling, W.T., 1988. *Numerical Recipes in C.* Cambridge University Press.

Shpitalni M. and Lipson H., 1996. Identification of Faces in a 2D Line Drawing Projection of a Wireframe Object. *IEEE Trans. Pattern–Analysis & Machine Intell.*. Vol. 18, No. 10, pp. 1000-1012.

Wang W. and Grinstein G., 1993. *A Survey of* 3D Solid Reconstruction from 2D Projection Line Drawings. *Computer Graphics Forum.* Vol. 12, No 2, pp. 137-158.