# Entropy of Scene Visibility

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**ABSTRACT**

We propose a new approach, based on information theory, to study the visibility of a scene. Thus, we will define the concepts of entropy and mutual information applied to 3D scene visibility. Mainly, we analize the concept of entropy (or randomness) of scene visibility and we examine the relationship between entropy of scene visibility and the expected value of the mean square error for all form factors. Next, these concepts are applied to diverse sample scenes and the accuracy of the values presented is analyzed.

**Key Words:** Rendering, Radiosity, Monte Carlo, Information Theory, Entropy

## 1 Introduction

In this paper, the visibility of a scene, which is directly related to form factors [1], is analyzed from the viewpoint of information theory. In many different fields, the concept of entropy has been studied at length and has been used as a starting point in order to study complexity [6, 10, 18, 19]. In our case, we study the entropy of scene visibility and leave the study of scene complexity for a future work. Also, this new approach, in a further research, will have to be applied to the field of radiosity and we hope that it will guide us in deciding on which is the best discretization between different scenes. In this context, we would like to remember Glassner's words [8]: "There has not been much attention paid to applying information theory to image synthesis. I think that it holds promise, though, and may help us design new types of efficient rendering algorithms".

The organization of this paper is as follows. In section 2, we present some concepts and results on form factors, random walk and information theory, which we will use throughout this paper. In section 3, information theory is applied to a scene and some relevant properties are studied. In section 4, we analize the concept of scene entropy or scene randomness. In section 5, we examine the relationship between scene entropy and expected value of the mean square error for all form factors. Finally, in section 6, our conclusions and future research are presented.

## 2 Previous concepts

### 2.1 Visibility and form factors

In the radiosity setting, a scene is composed by surfaces which are divided (discretized) into $n_p$ patches. The *form factor* $F_{ij}$ between patch $i$ and patch $j$ can be defined (for a planar patch $i$) as the fraction of lines that, exiting from $i$, have $j$ as the nearest patch intersected (thus $\sum_{j=1}^{n_p} F_{ij} = 1$) [16, 14]. In this way, $F_{ij}$ can be considered as the probability of a line that exiting from $i$ lands in $j$, and then the form factors from any patch form a probability vector. If we identify the lines connecting two patches with *visibility*, the form factor will thus give us the visibility between patches.

### 2.2 Random walk and form factors

Discrete *random walk* [13] is a Monte Carlo technique used to solve linear systems of equations. There are as many unknowns as states, and trajectories or paths are followed from state to state according to given transition probabilities. Each state has a corresponding transition probability vector, building up together a transition matrix. In the radiosity setting, where a state corresponds to a patch, the most common transition matrix used is the form factor matrix [17, 12, 5, 15], a matrix where the row $i$ is formed with the form factors from patch $i$. Although most of the time this matrix is not explicitly known, the probabilities it represents can be very easily simulated, and this is all that is needed to proceed with the random walk. As the transition probabilities we use only depend on the last visited state, we have a *Markov chain.*

A probability matrix is said to be aperiodic if it has no periodic state [3]. A periodic state is one such that can be visited back by a path starting from it only at multiples of the period. A probability matrix is said to be irreducible if there is always a path between any two states, that is, the probability of reaching any state starting at any other is not null. For the form factor matrix we can assume safely its aperiodicity. To assume its irreducibility every two patches must be reachable through a path from each other. However this is not always true: think for instance of two separated rooms with the interconnecting door closed, or wall patches behind a drawer, or floor patches covered by furniture. Nevertheless in the case of non-connecting rooms we can simply consider two different form factor matrices, one for each room. In the case of covered or hidden patches, we can simply suppress them and obtain a smaller and now irreducible form factor matrix. Some interesting properties can be proved by the form factor matrix. For the purpose of this paper we are mainly interested in the following two propositions [16]:

**Proposition 1** *If the form factor matrix $F$ is irreducible and aperiodic, then we have $\lim_{n\to\infty}(F^n)_{ij} \to \frac{A_j}{A_T}$ , where $A_j$ is the area of patch $j$ and $A_T = \sum_{i=1}^{n_p} A_i$, for all the $n_p$ patches in the scene.*

Thus, as the equilibrium distribution for a Markov chain is defined as the limit of the $n$th power of the transition matrix when $n$ grows to infinity, the above result means that we know the equilibrium distribution for the random walk corresponding to the form factors.

**Proposition 2** *When the length of a random path with transition matrix $F$ grows to infinity, the number of hits on any patch $i$ gets proportional to $\frac{A_i}{A_T}$, independently of where the path started its trajectory.*

This is simply a consequence of $\{\frac{A_i}{A_T}\}$ being the equilibrium distribution. A Markov chain with an equilibrium distribution is called ergodic. Thus, the form factors correspond to an ergodic Markov chain.

### 2.3 Information theory

We present here some basic results of information theory [2, 4]. The *Shannon entropy* $H$ of a discrete random variable $X$ with values in the set $\{a_1, a_2, \ldots, a_n\}$ is defined as

$$H(X) = -\sum_{i=1}^{n} p_i \log p_i \tag{1}$$

where $p_i = Pr[X = a_i]$, the logarithms are taken in base 2 and entropy is expressed in bits, and when $p_i = 0$ we take $p_i \log p_i = 0$. As $-\log p_i$ represents the *information* associated with the result $a_i$, the entropy gives the average information of a random variable. Thus, the entropy is a measure of the *uncertainty* or the ignorance of a random variable. For a given $X$, $H(X) \leq \log n$, with equality if and only if $p_i = 1/n$, for all $i$.

Let us consider another random variable $Y$ with probabilities $q_i$ corresponding to values in the set $\{b_1, b_2, \ldots, b_m\}$. The *joint entropy* of $X$ and $Y$ is defined as

$$H(X,Y) = -\sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij} \log p_{ij} \tag{2}$$

where $p_{ij} = Pr[X = a_i, Y = b_j]$. The *conditional entropy* is defined as

$$\begin{aligned} H(X|Y) &= \sum_{j=1}^{m} q_j H(X|Y = b_j) \\ &= -\sum_{j=1}^{m}\sum_{i=1}^{n} p_{ij} \log p_{i|j} \end{aligned} \tag{3}$$

where

$$H(X|Y = b_j) = -\sum_{i=1}^{n} p_{i|j} \log p_{i|j} \tag{4}$$

and $p_{i|j} = Pr[X = a_i|Y = b_j]$. The following relation holds between the different probabilities (Bayes theorem): $p_{ij} = q_j p_{i|j} = p_i p_{j|i}$. If $X$ and $Y$ are *independents*, we have $p_{ij} = p_i q_j$. The conditional entropy can be thought of in terms of a channel whose input is the random variable $X$ and whose output is the random variable $Y$. $H(X|Y)$ corresponds to the uncertainty in the channel input from the receiver's point of view.

The *mutual information* between two random variables $X$ and $Y$ is defined as

$$I(X,Y) = H(X) - H(X|Y) \tag{5}$$

From above definitions, we can obtain $I(X,Y) \geq 0$ and $I(Y,X) = H(Y) - H(Y|X) = I(X,Y)$. The mutual information represents the amount of information that one random variable, the output of the channel, gives about a second random variable, the input of the channel. $I(X,Y)$ is a measure of the shared information between $X$ and $Y$.

The *relative entropy* or *Kullback-Leibler distance* between two probability distributions $p =$

$\{p_i\}$ and $q = \{q_i\}$, that are defined over the same set $S = \{1, \ldots, n\}$, is defined as

$$D_{KL}(p\|q) = \sum_{i=1}^{n} p_i \log \frac{p_i}{q_i} \tag{6}$$

The relative entropy satisfies $D_{KL}(p\|q) \geq 0$ (Gibbs' inequality), with equality only if $p = q$. It is also known as *discrimination* and this is not strictly a distance. Moreover, we want to emphasize that the mutual information can be expressed as

$$I(X, Y) = D_{KL}(\{p_{ij}\}\|\{p_i q_j\}) \tag{7}$$

The joint entropy of $n$ random variables is defined as

$$\begin{aligned} H(X_1, \ldots, X_n) = H(X_1) + H(X_2|X_1) + \ldots \\ +H(X_n|X_1, \ldots, X_{n-1}) \end{aligned} \tag{8}$$

and if the $n$ variables are mutually independent:

$$H(X_1, \ldots, X_n) = \sum_{i=1}^{n} H(X_i) \tag{9}$$

The *entropy rate* or *entropy density* of a chain of random variables is given by

$$h = \lim_{n\to\infty} \frac{1}{n} H(X_1, X_2, \ldots, X_n) \tag{10}$$

and represents the average information content per output symbol. A Markov chain defined on a set $S$ of $n$ states can be described by a *probability transition matrix* $P_{j|i}$, which gives the probability of transition from any state $i \in S$ to any other state $j \in S$. A Markov chain can be viewed as a chain of random variables complying with

$$H(X_n|X_1, X_2, \ldots, X_{n-1}) = H(X_n|X_{n-1}) \tag{11}$$

An important result is the following theorem:

**Theorem 1** *An ergodic Markov chain has entropy rate or information content*

$$\begin{aligned} h &= \lim_{n\to\infty} \frac{1}{n} H(X_1, X_2, \ldots, X_n) \\ &= \lim_{n\to\infty} H(X_n|X_{n-1}) \\ &= H(Y|X) = \sum_{i=1}^{n} w_i H(Y|X = i) \\ &= -\sum_{i=1}^{n} w_i \sum_{j=1}^{n} P_{j|i} \log P_{j|i} \end{aligned} \tag{12}$$

*where $w_i$ is the equilibrium distribution.*

## 3 Scene entropy and other measures

In this section, the above results will be applied to a discretized scene. We consider a random walk in a discretized scene as a Markov chain where $P_{j|i} = F_{ij}$, $n = n_p$, $w_i = \frac{A_i}{A_T}$ and $X$,$Y$ are dependent random variables taking values over the set of patches $S = \{1, 2, \ldots, n_p\}$ with $\{\frac{A_i}{A_T}\}$ as probability distribution. Thus, we define the *entropy rate of scene visibility*, or simply *scene entropy*, as

$$\begin{aligned} H_s &= H(Y|X) = \sum_{i=1}^{n_p} \frac{A_i}{A_T} H(Y|X = i) \\ &= -\sum_{i=1}^{n_p} \frac{A_i}{A_T} \sum_{j=1}^{n_p} F_{ij} \log F_{ij} \end{aligned} \tag{13}$$

It is important to emphasize that the scene can be considered as a channel with input (source) $X$ and output (destination) $Y$, where $X$ and $Y$ take values over the set of patches with $\{\frac{A_i}{A_T}\}$ as probability distribution and the channel transition matrix is the form factor matrix. Thus, $H_s$ measures the average uncertainty that remains about the destination patch when the source patch is known. The scene entropy can also be interpreted as the area-weighted average of the entropies of all patches, where the *entropy of a patch $i$* is defined by

$$H_i = H(Y|X = i) = -\sum_{j=1}^{n_p} F_{ij} \log F_{ij} \tag{14}$$

and thus

$$H_s = \sum_{i=1}^{n_p} \frac{A_i}{A_T} H_i \tag{15}$$

Also, we define the *entropy of the ray position* as

$$H_p = H(X) = H(Y) = -\sum_{i=1}^{n_p} \frac{A_i}{A_T} \log \frac{A_i}{A_T} \tag{16}$$

and the *mutual information of a scene* is defined by

$$\begin{aligned} I_s &= I(X, Y) = H(Y) - H(Y|X) \\ &= -\sum_{i=1}^{n_p} \frac{A_i}{A_T} \log \frac{A_i}{A_T} + \sum_{i=1}^{n_p} \frac{A_i}{A_T} \sum_{j=1}^{n_p} F_{ij} \log F_{ij} \end{aligned} \tag{17}$$

In concordance with propositions 1 & 2 in section 2.2, the ray position entropy, $H_p$, may be interpreted as the uncertainty on the position (patch) of a particle travelling an infinite random walk. $H_p$ is the entropy of the equilibrium distribution. The mutual information, $I_s$, can be interpreted as the amount of information that the destination patch conveys about the source patch, and vice versa. Consequently, $I_s$ is a measure of the information transfer in a scene. On the other hand, $I_s$ can also be considered as a correlation measure of a scene [9, 18, 19].

Moreover, because of $H(X) = H(Y)$, we have $H_s = H(Y|X) = H(X|Y)$ and thus the symmetry or reversibility of the channel is shown. The Bayes theorem can be expressed by the property of the form factors

$$\frac{A_i}{A_T}F_{ij} = \frac{A_j}{A_T}F_{ji} \tag{18}$$

and the *joint entropy of a scene* is given by

$$\begin{aligned} H(X,Y) &= H_p + H_s = 2H_p - I_s = 2H_s + I_s \\ &= -\sum_{i=1}^{n_p}\sum_{j=1}^{n_p} \frac{A_i}{A_T}F_{ij} \log \frac{A_i}{A_T}F_{ij} \end{aligned} \tag{19}$$

This entropy can be interpreted as the uncertainty about the position and direction of a ray. It is the entropy of a random variable with probability distribution $\{\frac{A_i}{A_T}F_{ij}\}$.

Finally, we can observe that, when all areas are equal, we get $\frac{A_i}{A_T} = \frac{1}{n_p}$ and $F_{ij} = F_{ji}$, for all $i$ and $j$, and therefore

$$\begin{aligned} H_s &= H(Y|X) = -\frac{1}{n_p}\sum_{i=1}^{n_p}\sum_{j=1}^{n_p} F_{ij}\log F_{ij} \\ &= \frac{1}{n_p}\sum_{i=1}^{n_p} H_i \end{aligned} \tag{20}$$

and

$$H_p = H(X) = \log n_p \tag{21}$$

In this case, $H_s$ will get its maximum value when all $F_{ij}$ values are equal ($F_{ij} = \frac{1}{n_p}$): $H_s = H_p$. And the minimum value will be reached when all form factors from any patch are near zero except one with value near 1: $H_s \approx 0$.

## 4 Entropy and randomness of a scene

As we have seen in the previous section, scene entropy (entropy rate or entropy density) provides us with a general measure of the uncertainty or information content associated with a scene. Quoting Feldman's words [7], that we apply to a Markov chain, "the entropy density provides an answer to the question: in a Markov chain, given the knowledge of the previous symbol, how uncertain are you, on average, about the next symbol? Thus, we may view $h$ as the intrinsic unpredictability or the irreducible *randomness* associated with the chain".

An important result of information theory is that $H(X)$ gives a measure of the average length of the description of $X$ using an optimal binary coding (for example, Huffman coding). With a Huffman code, we can obtain an optimal coding for a random walk in a scene. Shannon entropy is strongly related to statistical entropy (Boltzmann, Gibbs) and also to Kolmogorov-Chaitin(KC) complexity (algorithmic randomness or algorithmic complexity) that can be used to measure disorder or randomness without any recourse to probabilities [11, 20]. The KC complexity of an object is defined as the length of the minimal Universal Turing Machine program needed to reproduce it. An important theorem says that the Shannon entropy of a random variable $X$ taking values in $S$ is equal, except for an additive constant, to the expected value of KC complexity of elements in $S$. Referring to Feldman once again [7]: "It turns out that $h$ is equal to the average length (per variable) of the minimal Turing machine program required to produce a typical configuration".

In a 3D scene context, it is especially interesting to ask about the extremal cases of maximum and minimum entropy, which correspond to the maximum disorder (randomness or unpredictability in the ray path) and maximum order (predictability), respectively. We must remark here that the concepts of order and disorder are not directly referred to the collocation of objects in space, but to visibility criteria. Maximum randomness is obtained in scenes with no privileged visibility directions, and maximum predictability in the contrary case.

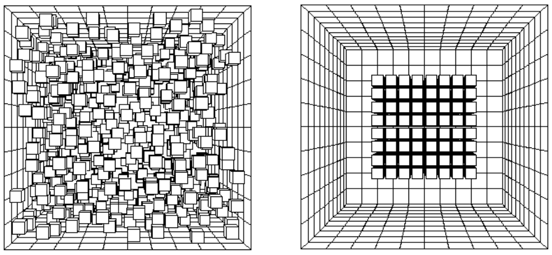
We illustrate both cases with the following examples:

*(i)* The maximum entropy is exemplified by the interior of an empty sphere divided into equal area patches. Here all form factors are equal and the uncertainty of the destination patch is maximum (no visibility direction is privileged). A scene of maximum entropy is called equilibrium or uniform scene. The information transfer is zero.

*(ii)* The minimum entropy is represented by a scene with almost touching objects, as the space between two very near concentrical spheres. There are strongly privileged visibility directions (for each patch one form factor is near 1). This system is highly correlated and the information transfer is maximum.

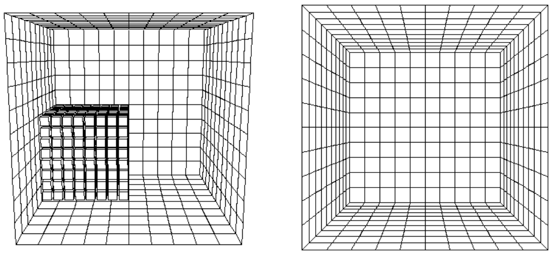
In scenes with the same discretization (as in table 1, where we have a cubical enclosure with 512 interior cubes), and consequently with the same $H_p$, where simply the interior objects have been displaced, we can observe that the increase of the entropy remains compensated for a mutual information decrease, and vice versa: more randomness less correlation (figure 1a), less randomness more correlation (figure 2a).

To be able to compare scenes with different dis-



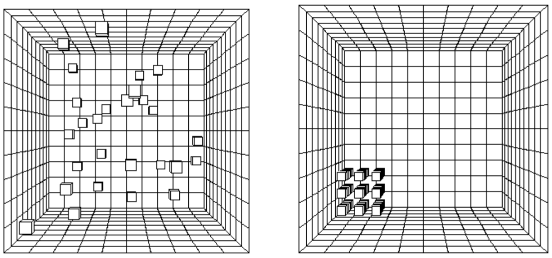
(a) (b)

Figure 1: (a) A random and (b) clustered configurations with 512 cubes.



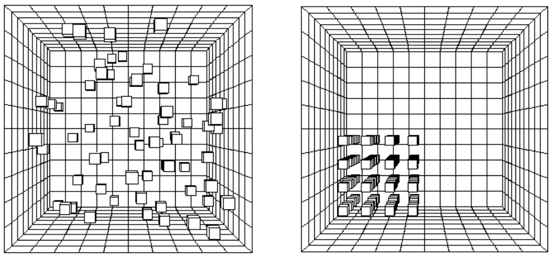
(a) (b)

Figure 2: (a) A clustered configuration with 512 cubes and (b) an empty cubical enclosure with its surfaces discretized into 10x10 patches.



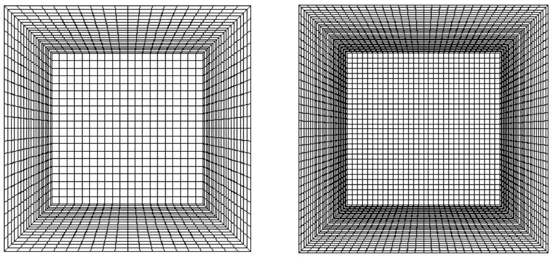
(a) (b)

Figure 3: (a) A random and (b) clustered configurations with 27 cubes.



(a) (b)

Figure 4: (a) A random and (b) clustered configurations with 64 cubes.



(a) (b)

Figure 5: Two empty cubical enclosures with their surfaces discretized into (a) 20x20 and (b) 30x30 patches.

| *Scene* | *Lines* | $H_s$ | $I_s$ | $E(MSE)$ |
|---|---|---|---|---|
| Fig.1a | $10^6$ | 6.370 | 5.171 | 1284.6 |
| Fig.1a | $10^7$ | 6.761 | 4.779 | 1286.7 |
| Fig.1b | $10^6$ | 5.072 | 6.469 | 949.3 |
| Fig.1b | $10^7$ | 5.271 | 6.270 | 950.1 |
| Fig.2a | $10^6$ | 4.674 | 6.867 | 898.8 |
| Fig.2a | $10^7$ | 4.849 | 6.692 | 900.9 |

Table 1: Results for the three cubical enclosures with 512 small cubes (figures 1a, 1b & 2a). The $E(MSE)$ is normalized to a single line. For each scene, $H_p = 11.541$ and $10^6$ and $10^7$ lines have been cast.

cretization, we can introduce the definition of *normalized scene entropy* as the quotient

$$\bar{H}_s = \frac{H_s}{H_p} \tag{22}$$

In table 2, where we start with an empty cubical enclosure and then we add small interior cubes, we can observe how the normalized entropy decreases when we introduce progressively more cubes. This fact increases the correlation in a scene.

How does scene entropy behave with an increase in the number of patches? According to information theory, when the size of the patches goes to zero, the scene entropy goes to infinity. This fact is partially illustrated in table 3 and figure 6, where we have a cubical enclosure with different discretizations of its surfaces (10x10, 20x20, 30x30).

## 5 Scene entropy and mean square error for all form factors

In this section, we want to analize the accuracy of the values presented in the different tables. The error in these values, which is manifested when the number of casting lines is increased, is directly

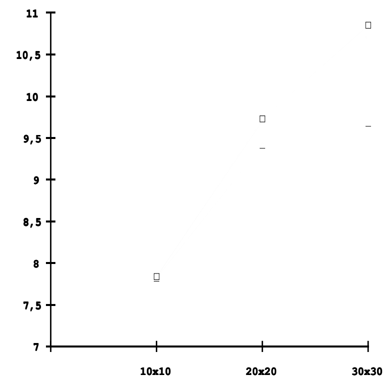



Figure 6: Scene entropy in vertical axis for the cubical enclosure with different discretitzations of its surfaces (figure 2b:10x10, figure 5a:20x20 and figure 5b:30x30). For each scene, $10^7$(triangles), $10^8$(diamonds) and $10^9$(squares) lines have been cast.

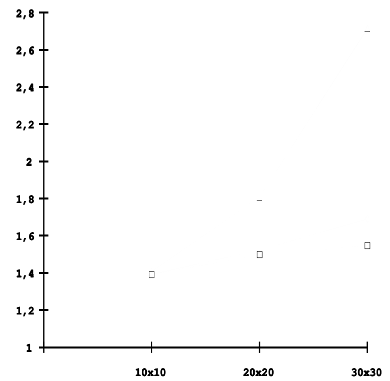



Figure 7: Mutual information in vertical axis for the cubical enclosure with different discretitzations of its surfaces (figure 2b:10x10, figure 5a:20x20 and figure 5b:30x30). For each scene, $10^7$(triangles), $10^8$(diamonds) and $10^9$(squares) lines have been cast.

| *Scene* | $H_s$ | $I_s$ | $\bar{H}_s$ |
|---|---|---|---|
| Fig.2b | 7.821 | 1.408 | 0.847 |
| Fig.3a | 7.780 | 1.669 | 0.823 |
| Fig.3b | 7.589 | 1.861 | 0.803 |
| Fig.4a | 7.709 | 2.009 | 0.793 |
| Fig.4b | 7.606 | 2.112 | 0.783 |
| Fig.1a | 6.761 | 4.779 | 0.586 |
| Fig.2a | 4.849 | 6.692 | 0.420 |

Table 2: Results for the empty scene and the scenes with 27, 64 and 512 small cubes (figures 2b, 3a, 3b, 4a, 4b, 1a & 2a). For each scene, $10^7$ lines have been cast.

| *Scene* | *Lines* | $H_s$ | $I_s$ | $E(MSE)$ |
|---|---|---|---|---|
| Fig.2b | $10^7$ | 7.821 | 1.408 | 497.7 |
| Fig.2b | $10^8$ | 7.837 | 1.392 | 497.7 |
| Fig.2b | $10^9$ | 7.838 | 1.391 | 497.7 |
| Fig.5a | $10^7$ | 9.420 | 1.809 | 2012.7 |
| Fig.5a | $10^8$ | 9.705 | 1.524 | 2013.0 |
| Fig.5a | $10^9$ | 9.731 | 1.498 | 2013.0 |
| Fig.5b | $10^7$ | 9.684 | 2.715 | 4543.9 |
| Fig.5b | $10^8$ | 10.708 | 1.691 | 4545.8 |
| Fig.5b | $10^9$ | 10.852 | 1.547 | 4546.1 |

Table 3: Results for the cubical enclosure with different discretitzations of its surfaces (figure 2b:10x10, figure 5a:20x20 and figure 5b:30x30). The $E(MSE)$ is normalized to a single line. For each scene, $10^7$, $10^8$ and $10^9$ lines have been cast.

related to error in form factors computation. The form factors have been computed with the integral geometry method described in [14, 16]. Next, we will study the relationship between the entropy and the mean square error of all form factors.

The variance for the form factor estimator [14, 16] is given by

$$Var(F_{ij}) = \frac{F_{ij}(1-F_{ij})}{N_i} \qquad (23)$$

where $N_i$ is the number of lines cast from patch $i$.

Let us consider now an importance vector $\alpha_i$, which gives us the importance we assign to each patch. For each patch $i$, we cast a number of lines, $N_i$, proportional to this importance, that is $N_i = \alpha_i N$, where $N$ is the total lines cast (a particular case would be $\alpha_i = \frac{A_i}{A_T}$). It is important to remember also that the variance is the expected value of the square error.

Let us suppose we consider now the expected value of the mean square error, weighted with the importance, for all form factors for all patches. As for each patch the expected value of the square error is the variance, and limiting the sum to the patches with $\alpha_i > 0$, we obtain:

$$\begin{aligned} E(MSE) &= \sum_i \alpha_i \sum_j Var(F_{ij}) \\ &= \sum_i \alpha_i \sum_j \frac{F_{ij}(1-F_{ij})}{N_i} \\ &= \sum_i \alpha_i \sum_j \frac{F_{ij}(1-F_{ij})}{\alpha_i N} \\ &= \frac{1}{N}(\sum_i \sum_j F_{ij} - \sum_i \sum_j F_{ij}^2) \\ &= \frac{1}{N}(n_p - \sum_i \sum_j F_{ij}^2) \end{aligned} \tag{24}$$

where $n_p$ is the number of patches in the scene. This expression has a minimum value when for all $i$ in the sum there exists a $j$ such that $F_{ij} \approx 1$ and $\approx 0$ otherwise. This ideal situation gives a null error, and obviously corresponds to the minimum entropy and maximum mutual information. On the other side, the maximum is obtained when all form factors are equal (with value $\frac{1}{n_p}$), which obviously corresponds to the maximum entropy.

The sets of scenes analyzed above can be used to study the relationship between scene entropy and the expected value of the mean square error for all form factors

In table 1, we can observe that the bigger the entropy the greater the error in form factors computation. This means that, for a given error, we need to cast more lines for a scene with more entropy. Also, by increasing the number of lines, we can see how the error in entropy estimation increases with the scene entropy. In tables 1 & 3 and figures 6 & 7, it is also shown that the increase in the number of lines increases the entropy estimation and, logically, decreases the mutual information estimation. The reason for this entropy increase is that in the algorithm used the form factors have an initial value of zero and so, with few lines, many form factors continue with the value of zero. This provoques an apparent bigger correlation in a scene, i.e., a lower value in entropy estimation and a higher value in mutual information estimation.

By increasing the number of patches, how does error behave? In table 3, we can observe how the error increases with the number of patches, in accordance with the increase of scene entropy.

## 6 Conclusions and future research

We have shown in this paper how the results of information theory can be applied to a 3D scene, to evaluate its entropy, mutual information and other measures. The results obtained in diverse sample scenes are in concordance with the theoretical analysis. That is, for scenes built up with the same discretization, the bigger the disorder the higher the entropy and lower mutual information, and the contrary for scenes with smaller disorder. We have also observed that the increase in the number of patches implies an increase in entropy. Equally, there exists a direct correspondence between entropy and error in form factors computation. We have seen that, for a given error, we need to cast more lines for a scene with more entropy.

Future work will be focused in two directions:

*(i)* We have seen that the *mutual information* can be defined as a discrimination with regard to independence ($p_{ij} = p_i q_j$). On the other hand, Solé, Blanc and Luque [19] have introduced a measure of complexity called *distance to independence.* Both measures will be studied as proposals for *visibility complexity* in a scene.

*(ii)* As we have already said in the introduction, information theory should be applied to the field of *radiosity* and we think that it will allow us to decide, with the help of the *mutual information*, on which is the best *discretization* between different scenes.

## 7 Acknowledgements

Many thanks to Roel Martínez and Joaquim Gelabertó. This project has been funded in part with grant number TIC98-0973-C03-03 of the Spanish Government.

## REFERENCES

[1] Esteve del Acebo, Miquel Feixas and Mateu Sbert. "Form Factors and Information Theory". *Proceedings of 3IA'98*, April 1998.

[2] Richard E. Blahut. *Principles and practice of Information Theory.* Addison-Wesley, 1987.

[3] Robert Coleman. *Stochastic processes.* George Allen&Unwin Ltd, London, 1974.

[4] T.M. Cover and J.A. Thomas. *Elements of Information Theory.* Wiley, 1991.

[5] Martin Feda and Werner Purgathofer. "Progressive ray refinement for Monte Carlo radiosity". *Fourth Eurographics Workshop on Rendering*, pp. 15–26, Paris, France, June 1993. Eurographics Technical Report Series EG 93 RW.

[6] David P. Feldman and James P. Crutchfield. "Statistical Measures of Complexity: Why?". *Physics Letters A*, 238:244–252, 1998.

[7] David P. Feldman. "A brief introduction to: Information Theory, Excess Entropy and Computational Mechanics". Department of Physics, University of California, July 1997. Available in *http://leopard.ucdavis.edu/dave/index.html.*

[8] Andrew S.Glassner. *Principles of digital image synthesis.* Morgan Kaufmann Publishers, San Francisco, 1995.

[9] W.Li. "Mutual information functions versus correlation functions". *J. Stat. Phys.*, 60(5/6):823-837, 1990.

[10] R. López-Ruiz, H.L. Mancini and X. Calbet. "A statistical measure of complexity". *Physics Letters A*, 209:321–326, 1995.

[11] Ming Li and Paul Vitányi. *An introduction to Kolmogorov Complexity and its applications.* 2nd ed., Springer-Verlag, New York, 1997.

[12] S. N. Pattanaik and S. P. Mudur. "Computation of global illumination by Monte Carlo simulation of the particle model of light". *Proceedings of the Third Eurographics Workshop on Rendering*, pp. 71–83, Bristol, UK, May 1992.

[13] Reuven Y. Rubinstein. *Simulation and the Monte Carlo method.* John Wiley & Sons, New York, 1981.

[14] Mateu Sbert. "An integral geometry based method for fast form factor computation". *Computer Graphics Forum (Eurographics '93)*, 12(3):C409–C420, September 1993.

[15] Mateu Sbert. "Error and complexity of random walk Monte Carlo radiosity". *IEEE Transactions on Visualization and Computer Graphics*, **3**(1), March 1997.

[16] Mateu Sbert. *The use of global random directions to compute radiosity. Global Monte Carlo methods.* Ph.D. thesis, Universitat Politècnica de Catalunya, Barcelona, March 1997. Available in *http://ima.udg.es/˜ mateu.*

[17] Peter Shirley. "A ray tracing method for illumination calculation in diffuse-specular scenes". *Proceedings of Graphics Interface '90*, pp. 205–212, Toronto, Ontario, May 1990.

[18] R. Solé and S.C.Manrubia. *Orden y caos en sistemas complejos.* Ediciones UPC, Barcelona, 1996.

[19] R. Solé, C.Blanc and B.Luque. *Statistical Measures of Complexity for Strongly Interacting Systems.* Santa Fe Institute Working Paper 97-11-083, 1997.

[20] W.H. Zurek. "Algorithmic randomness and physical entropy". *Physical Review A*, 40(8):4731–4751, 1989.