CLOTH DEFORMATION MODELLING USING A PLATE BENDING MODEL

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ABSTRACT

Modelling and simulation of cloth deformation are of great importance in computer graphics and engineering applications. Although many models have been developed, problems still exist. No model so far can accurately describe both motion and deformation of cloth in a unified equation. And the equations for the in-plane deformation of cloth have yet to be developed. In this paper, a model for both motion and deformation of cloth will be developed which will account for the in-plane deformation of cloth. In this model, the cloth deformations under out-plane and in-plane loads were uncoupled into out-plane and in-pane deformations. The theory of small deflection bending of plates with elastic behaviours was applied to the out-plane deformation and the theory of plane stress of elastic objects was applied to the in-plane deformation. Both static and dynamic problems of cloth were taken into account. The basic governing equations were given and the corresponding finite difference formulae were developed. Two examples were given to demonstrate the applications of the proposed approach.

Keywords: cloth deformation, plate bending, computer graphics, computer simulation and animation.

INTRODUCTION

Simulation of cloth deformation is an important subject in computer graphics, computer animation and the textile industry. Because of its academic and practical significance, researchers from all over the world have devoted a good deal of effort to it and proposed various simulation methods.

However, despite the plethora of proposed methods, cloth drape simulation still proves to be a difficult problem in practice. More work is still needed in order to improve its accuracy, computational efficiency and generality. For example, motion and deformation of cloth need to be addressed accurately in a unified equation. The models developed from the energy minimisation and finite element methods only considered the deformation of cloth. In the models developed from Lagrange's equations of motion, both motion and deformation of the cloth are taken into account. Nevertheless, the deformation of the cloth was determined by calculating an energy function of a deformable curve that has no direct relation to the physical and mechanical properties of cloth. Therefore, it is not capable of accurate modelling of cloth deformation.

To solve these problems, we propose a new method to simulate the deformation of cloth. In our method, both motion and deformation of the cloth are treated in a set of unified equations. These equations are developed from elasticity, closely relating to the elastic properties of the cloth, and the theories of plates. Both in-plane and out-plane deformations are considered in our model. A finite difference technique is employed in solving the mechanics equations.

LITERATURE REVIEW

Generally speaking, there are two categories of techniques for the simulation of cloth deformation: the geometric and the physically based approaches.

The geometric techniques are usually less expensive, but do not take the physical and mechanical properties of cloth into account. Weil [Weil86] represented a hanging cloth as a grid of points. The cloth's shape was represented by fitting catenary curves between the hanging points and the constrained points. Agui, Nagao and Nakajma [Agui90] used a hollow cylinder consisting of a series of circular rings to model a sleeve on a bending arm. Hinds and McCartney [Hinds90] defined 3-D garment panels as surfaces with respect to an underlying body and applied this method to interactive garment design. This technique aims to provide an animation tool that could quickly produce images of clothed objects.

Since the geometric techniques cannot simulate the complex shapes of cloth deformation realistically, much more work has focused on the physically based techniques. In these techniques, some models were based on the motion equations of forces, some on the energy equations and others were developed directly from finite element methods.

Most approaches were derived from the use of the motion equations. To develop a deformable model for generalised flexible objects, Terzopoulos et al. [Terzo88] employed the Lagrange equations of motion. In their model a finite difference or finite element method can be used to solve the dynamic problems of cloth. Late, this model was used by Thalmann et al. [Thalm96] in animating clothes on synthetic actresses. Li et al. [Li96] considered a particular problem where a piece of cloth is suspended in airflow. In their work, the simplified Reynold's averaged Navier-Stokes equation was used to determine the forces on each patch and the forces on the whole cloth. Cloth deformation was determined by considering these air forces. Aono [Aono90] developed a wrinkle-propagation model based upon the 3-D kinetic differential equations of elasticity. In this model, isotropy, anisotropy and viscoelasticity were considered and the finite difference method was applied to solve the equations. Volino et al. [Volin96] applied Newton's law of motion to simulate the cloth movement and deformation. The equations were solved using the second order Euler-Cromer method. Sakaguchi et al. [Sakag95] developed a dynamically deformable cloth model based on Newton's law of motion considering the damping effect of cloth. Provot [Provo95] proposed a classic mass-spring model and Newton's law of motion to simulate cloth under constraints. Breen et al. [Breen92] used particles to model the draping behaviour of woven cloth. The crossing points of the warp and weft threads were treated as particles. Initially, the positions of these particles were determined using Newton's law of motion and taking into account the viscous drag resistance of air. Next, the total energy of the particles was minimised to generate the fine detail in the shape of the cloth. The total energy consisted of the repulsion energy among particles, the stretching energy, the energy due to inter-thread bending out of the plane, the energy due to inter-thread bending in the plane and the gravitational energy.

The rest shape of a piece of cloth is the consequence of energy minimisation. This fact lays the theoretical foundation of many other techniques in cloth simulation. Feynman [Feynm86] derived an energy equation of cloth from the theory of elastic plates which includes elastic energy, bending energy and gravitational energy. The steepest descent method is used in minimising the overall energy. This energy equation was also used by Ng. et al. [Ng95] to determine the cloth's shape. An improved multigriding method and searching method for the energy minimum allowed the system to run on a PC. Okabe et al. [Okabe92] considered the elongation energy, the shearing energy, the bending energy, the twisting energy, the gravitational energy and the penalty energy of cloth in their energy equation. Eberhardt et al. [Eberh96] introduced the Lagrange equations for a holonomic dynamic system and considered the kinetic energy, the potential energy, the tension energy, the shearing energy and the bending energy of a particle system for the modelling of cloth draping.

Finite element computations, as a general analysis tool in engineering, have also found their applications in the simulation of cloth. For example, Gan et al. [Gan95] analysed the fabric deformation characterised by large displacements and rotations with small strains using geometric non-linear shell / plate finite elements. Chen et al. [Chen95] introduced the shear flexible shell theory to predict the drape of fabric. In this model, the fabric is considered to be a continuous, orthotropic medium. Eischen et al. [Eisch96] proposed a finite element method based on non-linear shell theory to simulate 3D motions related to real fabric manufacturing processes. They considered fabric contact with rigid surfaces. Ascough et al. [Ascou96] simulated the behaviour of cloth draped about a human body form by modelling a garment as a mesh of simple beam finite elements, with geometric stiffness to account for large displacements. Zhao et al. [Zhao97] developed a model to simulate cloth deformation in three-dimensional space from two-dimensional patterns by using the finite element method of thin plate bending with geometric constraints.

OUT-PLANE DEFORMATION

Cloth or a garment is subject to both in-plane and out-plane forces resulting in an in-plane and an out-plane deformation. Here, by in-plane forces we mean that the directions of the forces are within the plane and by out-plane forces we mean that the directions of the forces are perpendicular to the plane of the cloth. Similarly, in-plane deformations denote the displacements which are within the plane and an out-plane deformation represents the displacement that is perpendicular to the cloth surface. Due to the properties of most cloth materials, the in-plane deformations are so small that the in-plane force and out-plane force can be uncoupled. Therefore, these two deformations can be formulated separately.

Bending deformation is usually the main form of deformation. Under out-plane loads, the strain-displacement relationship is given by

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} \\ \varepsilon_y &= \frac{\partial v}{\partial y} = -z\frac{\partial^2 w}{\partial y^2} \\ \gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = -2z\frac{\partial^2 w}{\partial x \partial y} \end{aligned} \tag{1}$$

Where u, v and w are displacements and ε_x, ε_y and γ_{xy} are strains along x, y and z directions respectively.

For an isotropic model, the relationship between stresses and strains observes the generalised Hooke's law that has the form

$$\begin{aligned} \sigma_x &= \frac{E}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_y) \\ \sigma_y &= \frac{E}{1-\mu^2}(\varepsilon_y + \mu\varepsilon_x) \\ \tau_{xy} &= \frac{E}{2(1+\mu)}\gamma_{xy} \end{aligned} \tag{2}$$

Where σ_x, σ_y and τ_{xy} are stresses; E is Young's modulus and μ is Poisson's ratio.

Substitution of Eq (1) into (2) gives the stress-deflection relationship

$$\begin{aligned} \sigma_x &= -\frac{Ez}{1-\mu^2}\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right) \\ \sigma_y &= -\frac{Ez}{1-\mu^2}\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right) \\ \tau_{xy} &= -\frac{Ez}{1+\mu}\frac{\partial^2 w}{\partial x \partial y} \end{aligned} \tag{3}$$

For static problems of cloth bending, the basic partial differential equation is derived from the variation of the overall energy of cloth bending. That is

$$\Delta U = \Delta W \tag{4}$$

Where U is the overall strain energy, which can be written as

$$U = \frac{1}{2}\int_V (\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy})dV \tag{5}$$

and W is the overall work contributed by the out-plane load taking the form of

$$W = \int_S qwdS \tag{6}$$

Where q is the out-plane load.

Substituting Eqs (1) and (3) into (5), and then substituting Eqs (5) and (6) into (4), we obtain the following fourth order partial differential equation for the bending of the cloth

$$D\nabla^4 w = q \tag{7}$$

Where

$$\begin{aligned} D &= \frac{Eh^3}{12(1-\mu)^2} \\ \nabla^4 &= \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \end{aligned} \tag{8}$$

Where h is the thickness of the cloth.

For dynamic problems of cloth bending, the dynamic equilibrium condition can be written as follows

$$q_s + q_i + q_d = q \tag{9}$$

Where q_s is the elastic deformation force of the cloth, q_i is the inertial force, q_d is the damping force and q is the applied external force.

For isotropic elastic objects, according to Eq (9), the dynamic equilibrium equation has the form of

$$D\nabla^4 w + \overline{m}\frac{\partial^2 w}{\partial t^2} + c_z \frac{\partial w}{\partial t} = q \qquad (10)$$

Where $\overline{m}$ is the density and c_z is the damping coefficient of the cloth in the z-direction.

From Eq. (10), it can be seen that both motion and deformation are addressed in a unified model.

IN-PLANE DEFORMATION

In-plane deformation plays an important role in the modelling of stretchable fabrics such as sweaters. Under the in-plane loads, the in-plane deformation can be characterised using the basic equations of plane stress. The strain-displacement relation for the plane stress takes the following form

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} \\ \varepsilon_y &= \frac{\partial v}{\partial y} \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{aligned} \qquad (11)$$

The stress-strain relation is given by Eq (2) for isotropic elastic objects and the equilibrium equation for the static plane problems is given by

$$\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + X = 0 \\ \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + Y = 0 \end{aligned} \qquad (12)$$

Where X and Y are volumetric forces in the x and y directions respectively.

Substituting Eq (11) into (2), and then substituting Eq (2) into (12), we obtain the equilibrium equation with respect to displacements u and v for isotropic elastic objects

$$\begin{aligned} \frac{E}{1-\mu^2}\left(\frac{\partial^2 u}{\partial x^2} + \frac{1-\mu}{2}\frac{\partial^2 u}{\partial y^2} + \frac{1+\mu}{2}\frac{\partial^2 v}{\partial x \partial y}\right) + X = 0 \\ \frac{E}{1-\mu^2}\left(\frac{\partial^2 v}{\partial y^2} + \frac{1-\mu}{2}\frac{\partial^2 v}{\partial x^2} + \frac{1+\mu}{2}\frac{\partial^2 u}{\partial x \partial y}\right) + Y = 0 \end{aligned} \qquad (13)$$

For dynamic problems, the equilibrium equation will be replaced by the kinetic equation of plane problems, which has the form of

$$\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + X = \overline{m}\frac{\partial^2 u}{\partial t^2} + c_x \frac{\partial u}{\partial t} \\ \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + Y = \overline{m}\frac{\partial^2 v}{\partial t^2} + c_y \frac{\partial v}{\partial t} \end{aligned} \qquad (14)$$

Where c_x, c_y are the damping coefficients in the x and y directions, respectively.

Substituting the stress-strain relation (2) of isotropic elastic objects into (14), and then substituting the strain-displacement relation (11) into (14), we obtain the following kinetic equations

$$\begin{aligned} &\frac{E}{1-\mu^2}\left(\frac{\partial^2 u}{\partial x^2} + \frac{1-\mu}{2}\frac{\partial^2 u}{\partial y^2} + \frac{1+\mu}{2}\frac{\partial^2 v}{\partial x \partial y}\right) + X = \\ &\overline{m}\frac{\partial^2 u}{\partial t^2} + c_x \frac{\partial u}{\partial t} \\ &\frac{E}{1-\mu^2}\left(\frac{\partial^2 v}{\partial y^2} + \frac{1-\mu}{2}\frac{\partial^2 v}{\partial x^2} + \frac{1+\mu}{2}\frac{\partial^2 u}{\partial x \partial y}\right) + Y = \\ &\overline{m}\frac{\partial^2 v}{\partial t^2} + c_y \frac{\partial v}{\partial t} \end{aligned} \qquad (15)$$

FINITE DIFFERENCE APPROXIMATION

Once the above equations have been derived, the next step is to develop the resolving algorithm. In order to have a direct control over the resolving process; a finite difference method is deployed, which provides the finite difference approximation for Eqs (7), (10), (13) and (15).

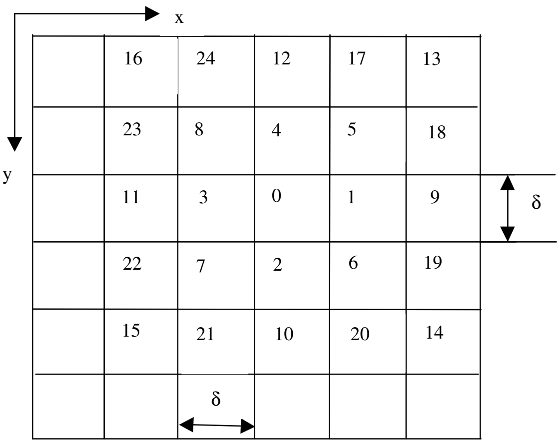

Fig. 1 Typical nodes of the difference method

Basic finite difference formulae have the forms of central difference, forward difference and backward difference. Among them central difference is widely used. With reference to the mesh and typical node representation shown in Fig. 1, we can develop the first to fourth order central difference formulae at node 0. Applying these formulae to the static out-plane deformation of cloth, Eq (7), it can be written as

$$20w_0 - 8(w_1 + w_2 + w_3 + w_4) + 2(w_5 + w_6 + w_7 + w_8) + (w_9 + w_{10} + w_{11} + w_{12}) = \frac{q_0 \delta^4}{D} \quad (16)$$

For the dynamic out-plane deformation of cloth, the finite difference formulae of the governing equation (10) has the form of

$$\frac{D}{\delta^4}[20w_0^t - 8(w_1^t + w_2^t + w_3^t + w_4^t) + 2(w_5^t + w_6^t + w_7^t + w_8^t) + (w_9^t + w_{10}^t + w_{11}^t + w_{12}^t)] + \frac{1}{\Delta t}\left(\frac{\overline{m}}{\Delta t} + \frac{c_z}{2}\right) w_0^{t+\Delta t} - \frac{2\overline{m}}{\Delta t^2} w_0^t + \frac{1}{\Delta t}\left(\frac{\overline{m}}{\Delta t} - \frac{c_z}{2}\right) w_0^{t-\Delta t} = q_0^t \quad (17)$$

Applying the same basic finite difference formulae, Eq (13) that describes the static in-plane deformation of cloth is given by

$$\begin{aligned} &u_1 + u_3 + \frac{1-\mu}{2}(u_2 + u_4) - (3-\mu)u_0 + \frac{1+\mu}{8}(v_6 + v_8 - v_5 - v_7) + \frac{1-\mu^2}{E}\delta^2 X_0 = 0 \\ &v_2 + v_4 + \frac{1-\mu}{2}(v_1 + v_3) - (3-\mu)v_0 + \frac{1+\mu}{8}(u_6 + u_8 - u_5 - u_7) + \frac{1-\mu^2}{E}\delta^2 Y_0 = 0 \end{aligned} \quad (18)$$

For the dynamic in-plane deformation of cloth, the finite difference formula of Eq (15) is derived as

$$\begin{aligned} &\frac{E}{(1-\mu^2)\delta^2}\left[u_1^t + u_3^t + \frac{1-\mu}{2}(u_2^t + u_4^t) - (3-\mu)u_0^t + \frac{1+\mu}{8}(v_6^t + v_8^t - v_5^t - v_7^t)\right] - \\ &\frac{1}{\Delta t}\left(\frac{\overline{m}}{\Delta t} + \frac{c_x}{2}\right)u_0^{t+\Delta t} + \frac{2\overline{m}}{\Delta t^2}u_0^t - \frac{1}{\Delta t}\left(\frac{\overline{m}}{\Delta t} - \frac{c_x}{2}\right)u_0^{t-\Delta t} + X_0^t = 0 \\ \\ &\frac{E}{(1-\mu^2)\delta^2}\left[v_2^t + v_4^t + \frac{1-\mu}{2}(v_1^t + v_3^t) - (3-\mu)v_0^t + \frac{1+\mu}{8}(u_6^t + u_8^t - u_5^t - u_7^t)\right] - \\ &\frac{1}{\Delta t}\left(\frac{\overline{m}}{\Delta t} + \frac{c_y}{2}\right)v_0^{t+\Delta t} + \frac{2\overline{m}}{\Delta t^2}v_0^t - \frac{1}{\Delta t}\left(\frac{\overline{m}}{\Delta t} - \frac{c_y}{2}\right)v_0^{t-\Delta t} + Y_0^t = 0 \end{aligned} \quad (19)$$

APPLICATIONS

As a demonstration of the proposed method, we will present two simple examples of cloth simulation. In both examples the above formulae were used to compute the deformed geometric models. The computations were performed on a Silicon Graphics O2 workstation. The geometric models were subsequently were textured and rendered using the commercial animation package Maya.

In our first example we simulate a square tablecloth which is hang from its four corners. The tablecloth is initially flat and located at the z = 0 plane. Fig 2a depicts the tablecloth in its neutral position. The tablecloth is then subjected to a uniformly distributed gravity force in the z-direction. To compute the finite differences across the tablecloth, its surface is discretized into a mesh of 9×9 nodal points. The final deformed shape of the tablecloth is obtained by taking into account the displacement of all the nodal points. Fig. 2b depicts the shape of the tablecloth after the deformation.

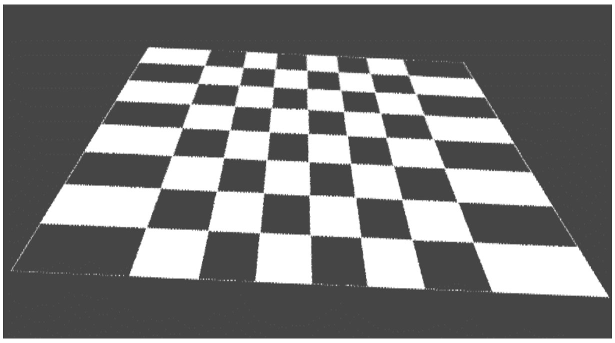

Fig. 2 (a) The tablecloth before the deformation.

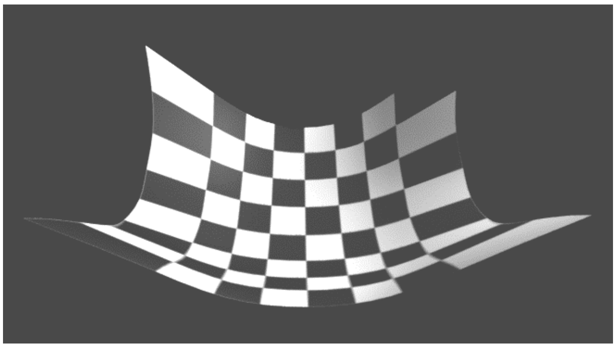

Fig. 2 (b) The tablecloth hanging from its corners after the deformation.

In our second example we simulate a square tablecloth resting on a square table and draping from its edges. Initially the tablecloth is allowed to rest on the table in its neutral position (Fig. 3a) and then we let gravity take effect (Fig 3b). The tablecloth surface is again meshed into 11 × 11 nodal points. A constraint is set such that the nodes resting on the tabletop are prevented from moving during the simulation. All other nodes are subjected to gravity and force the tablecloth to drape over the edges of the table.

Fig. 3 (a) The tablecloth before the deformation.

Fig. 3 (b) The tablecloth after the deformation.

CONCLUSIONS

Simulation of cloth deformation has many applications in engineering, the textile industry and the entertainment industry. It involves both computer graphics and more importantly the understanding of the physical properties of such fabric materials.

Cloth and many textile materials are those which exhibit great out-plane deformation, but very little in-plane or stretching deformation. This observation makes it justifiable to uncouple the out-plane and in-plane deformation components in the derivation of the governing equations, which greatly simplifies the formulation and the resolution process. As a consequence, computational efficiency is improved.

The basis of the proposed approach is the theory of plate bending with a small deflection. Both static and dynamic problems of cloth were considered and a set of unified equations derived that take into account both in-plane and out-plane deformations. The solution was computed using the finite difference method. The two simple examples, presented to demonstrate the validity of the proposed approach, were implemented on a SiliconGraphics O2 workstation.

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