AN AUTOMATIC ESTIMATION OF THE AXIS OF ROTATION OF FRAGMENTS OF ARCHAEOLOGICAL POTTERY: A MULTI-STEP MODEL-BASED APPROACH

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ABSTRACT

This paper presents a method for an estimation of the axis of rotation of radially symmetric objects from their fragments. In particular, fragments of archaeological pottery are considered. The method exploits basic geometrical properties of fragments which are described by an appropriate model. The estimation is performed in several steps which continuously improve accuracy and robustness of an initial estimate. Because only coordinates of surface points of a fragment are required for the estimation, the proposed method can be implemented as a fully automatic process. The approach has been evaluated on both synthetic objects and real pottery fragments.

Keywords: radial symmetry, axis of rotation, least squares optimization, M-estimators, robust fitting

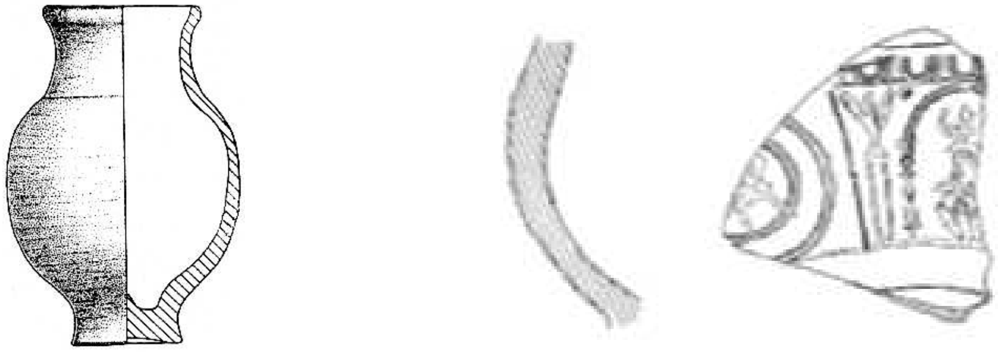
INTRODUCTION

One of the most important archaeological objects are fragments of ancient pottery. All the fragments found at excavation sites are measured, classified and archived. Until now archaeologists have classified fragments manually, which represents a great deal of routine work and takes much time (from 1/2 to 3 hours for each fragment). In order to help archaeologists in that task, several projects of a computer aided classification of pottery fragments were introduced in [Lübbe89], [Steck89] and [Sabla96].

For the classification, various shape and material characteristics of fragments are estimated [Menar97]. Because the investigated pottery were made on a potter's wheel, the original pots are assumed to be radially symmetric. Regarding the symmetry, a shape of a pot can be characterized by its *profile*, that means by the intersection of the pot with the plane going through its axis of rotation. An example of a profile of a pot is depicted in Fig. 1(a). The same characterization of a shape is also used for pottery fragments as can be seen in Fig. 1(b). Based on their profiles, fragments are divided into different prototype categories during the classification. The profiles also play an important role when original pots are reconstructed from their fragments.

In order to estimate the profile of a particular fragment, its *axis of rotation* has to be determined first. A position of the axis can be provided by archaeologists as a priori expert knowledge. But, regarding the huge number of fragments (several thousands), the estimation should be done automatically. For the automatic estimation of the axis, two different approaches can be found in literature. Both methods exploit the properties of surface normals of radially symmetric objects. The normals are estimated from range images of fragments. Ben Yacoub and Menard [Yacou97] proposed a robust estimation based on a *Hough transform*. The method is very stable against noise and outliers in the range data, but it lacks in speed and accuracy. The alternative *optimization approach* described by Halíř [Halíř97b] produces more accurate results, but due to the least squares minimization used it is not robust enough.

In this paper we propose a *multi-step approach* based on a model of a fragment of archaeological pottery. All the estimation steps exploit basic geometrical properties of the fragment which are independent on its position. The steps are organized in such a way that a good approximation of the rotation axis can be obtained even in the first step. If the solution is not accurate enough, the subsequent steps are applied to further improve the estimate. Finally, after completion of all the steps, a robust and precise estimate of the axis is provided.



(a) (b)

Figure 1: Characterization of a shape of an archaeological pottery by a profile: (a) a whole pot; (b) a pottery fragment

A MULTI-STEP MODEL-BASED ESTIMATION OF THE AXIS OF ROTATION

The estimation process consists of the following steps:

1. an estimation of an initial position the rotation axis by a *direct least squares optimization*
2. a robust estimation based on *M-estimators*
3. an *iterative refinement* of the estimated rotation axis by a *robust circle and line fitting*

All the steps are based on a model of a pottery fragment. In this section, the model of a fragment and its geometrical properties are presented first. Then, the individual estimation steps are discussed.

A Model of a Pottery Fragment

During the classification of a pottery fragments, archaeologists assume that original pots were made on a potter's wheel. Regarding this constraint, ancient pots can be modeled as *radially symmetric objects*. Such objects and its fragments have two important geometrical properties demonstrated in Fig. 2:

a) normals of the object/fragment surface go through the axis of rotation

b) a plane perpendicular to the rotation axis intersects the object in a circle and the fragment in a circular arc with the same center lying on the axis

Under the assumption that the original pots were radially symmetric, fragments of archaeological pottery can be described by the same model. The geometrical properties of a fragment do not depend on its position, thus they can be used for an estimation of an appropriate axis of rotation. And this is the main idea of the estimation process proposed in the following sections.

An Estimation of an Initial Position of the Axis of Rotation

In the beginning of the estimation process, an initial position of the axis of rotation is determined. The estimation assumes radial symmetry of ancients pots and exploits the first property of radially symmetric objects: *surface normals go through the axis of rotation of the object*. There exist two different approaches based on this fact: Hough transform [Yacou97] and numerical optimization [Halíř97b]. The Hough transform inspired method is robust against outliers, but it lacks in speed and accuracy. Regarding that, an *optimization approach* is used for the initial estimation.

The initial position of the axis is obtained using a standard least squares approach as the line a which minimizes the following objective function:

$$\min_a \sum_{i=1}^{M} d^2(n_i, a) \ , \tag{1}$$

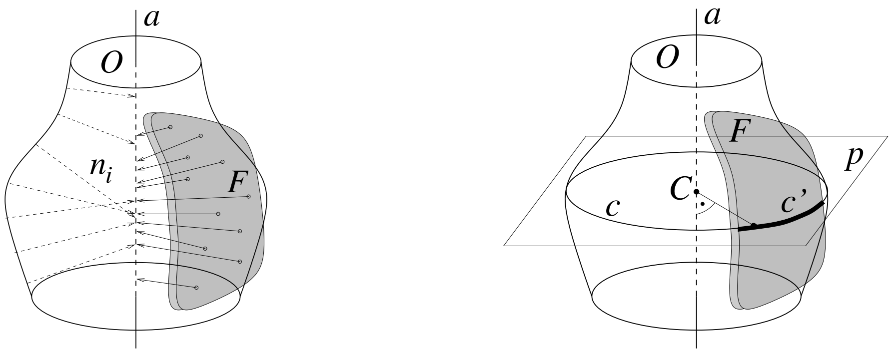
where M is the number of surface normals and $d(n_i, a)$ is the Eucleidan distance between the normal n_i and the axis a. The distance can be expressed as the length of the transversal line between the particular normal $n_i = X_i + t_i \cdot N_i$ given by the point X_i and the normal direction N_i, and the rotation axis $a = X_0 + t_0 \cdot N_0$ given by the point X_0 and the normal direction N_0:

$$d(n_i, a) = \frac{(X_i - X_0) \odot (N_i \times N_0)}{\|N_i \times N_0\|} \ , \tag{2}$$

where the operators $\odot$ and $\times$ denote dot and vector product of two vectors, respectively, and $\|.\|$ is the length of a vector. If the normals N_i and N_0 are parallel, Eq. 2 is reduced to

$$d(n_i, a) = \|(X_i - X_0) \times N_0\| \ . \tag{3}$$

The optimization problem Eq. 1 is solved for the unknown rotation axis a given by the point X_0 and the normal direction N_0. It can be shown that the optimal position of the point X_0 depends on the direc-



(a) (b)

Figure 2: A model of a radially symmetric object O with the axis of rotation a and its fragment F: a) surface normals n_i go through the axis a; b) a plane p perpendicular to the axis a intersects the object O in a circle c and the fragment F in a circular arc c' with the same center C lying on the axis

tion N_0. Regarding that, an initial position of the axis of rotation can be determined by an optimization process which runs over only two-dimensional searching space. Such estimation (which was originally proposed in [Halíř97b]) is effective, fast and reliable.

A Robust Estimation of the Axis of Rotation

In the initial estimation step, a position of the axis of rotation is estimated by a direct least squares minimization. This type of optimization approaches is known to be sensitive to outliers. As pointed out in [Barne84], with a least squares technique, *even one or two outliers in a large set can wreak havoc*! Regarding that, statisticians have developed various sorts of robust statistical estimators. The most relevant class for an estimation of parameters is so called *M-estimators*.

Mathematical theory of M-estimators is available in many books, see for example [Press93] or [Rey83]. The basic idea of robust estimation is to reduce the influence of outliers by replacing the squared residuals r_i^2 in the standard least squares minimization

$$\min \sum_i r_i^2 \tag{4}$$

by another less increasing function ρ of the residuals, yielding

$$\min \sum_i \rho(r_i) \ . \tag{5}$$

Instead of solving Eq. 5 directly, the minimization can be implemented as an iterative re-weighted process

$$\min \sum_i w(r_i^{(k-1)})\, r_i^2 \ , \tag{6}$$

where w is so called *weight function* and the superscript $^{(k)}$ indicates the iteration number. There exist a wide range of weight functions with different properties, the ones suitable for parameter estimation are discussed in [Zhang95, chapter 9]. A very popular among statisticians is *Huber's function*

$$w(r_i) = \begin{cases} 1 & \text{if} \quad |r_i| \le c\hat{\sigma} \\ \dfrac{c\hat{\sigma}}{|r_i|} & \text{otherwise} \end{cases} \ , \tag{7}$$

where $\hat{\sigma}$ is a robust standard deviation of residual errors r_i and c is a tuning constant with value $c = 1.345$. Another widely used weight function is *Tukey's biweight*

$$w(r_i) = \begin{cases} \left[1 - \left(\dfrac{|r_i|}{c\hat{\sigma}}\right)^2\right]^2 & \text{if} \quad |r_i| \le c\hat{\sigma} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

with the tuning constant $c = 4.6851$. The robust standard deviation $\hat{\sigma}$ of residual errors r_i can be estimated as [Barne84]

$$\hat{\sigma} = 1.4826 \operatorname*{median}_i |r_i| \ . \tag{9}$$

An application of the M-estimators for a robust determination of the axis of rotation is straightforward and it leads to the following algorithm:

1. get an initial position of the axis $a = X_0 + t_0 \cdot N_0$ by the direct least squares minimization of Eq. 1 with distances given by Eq. 2 and Eq. 3
2. use distances from the currently estimated axis a as residua $r_i = d(n_i, a)$
3. estimate the robust standard deviation $\hat{\sigma}$ of the residua r_i (Eq. 9)
4. compute weights $w(r_i)$ from residua r_i by the chosen weight function w (for example Eq. 7 or Eq. 8)
5. estimate a new position of the axis $\tilde{a}$ which minimizes the weighted least squares problem Eq. 6
6. convergence check: if $\|\tilde{a} - a\| > \varepsilon$, set $a = \tilde{a}$ and go back to the step 2

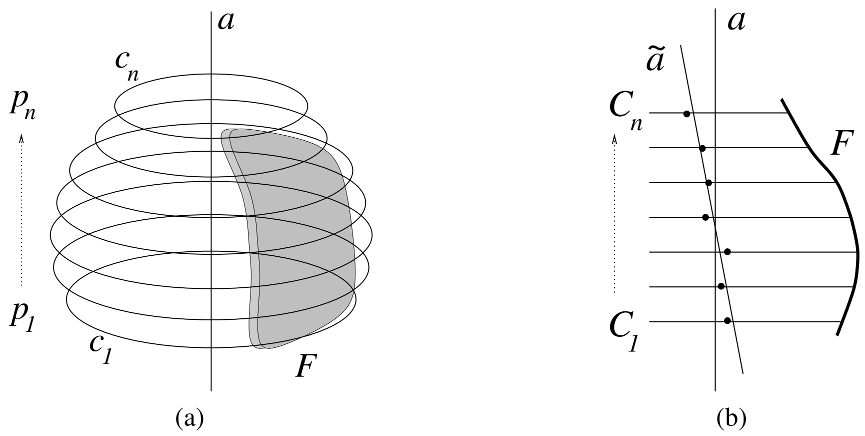


Figure 3: An estimation of the axis of rotation by a circle/line fitting; a) circle fitting, b) line fitting; see text for more details

As can be seen, the algorithm is an iterative re-weighted modification of the initial direct least squares minimization. Regarding that, it can be implemented very easily on top of the original estimation. The application of M-estimators preserves efficiency and reliability of the computation and adds robustness against outliers in the data and stability to systematic errors during processing of fragments.

An Iterative Refinement of the Estimated Rotation Axis

The previous estimation steps are based on property of surface normals. The problem is that the normals are typically not known in advance and they have to be determined first. During the determination, various factors (such as noise, outliers and systematic errors) degrade estimates of the normals and, consequently, the whole estimation of the axis of rotation. Regarding that, the position of the axis can be further improved by another, non-normal based, estimation method.

To estimate the axis of rotation of radially symmetric objects without knowledge on their surface normals, the second property of such objects can be exploited: *a plane perpendicular to the rotation axis intersects the object in a circle and its fragment in a circular arc with the same center lying on the axis*. This idea was firstly used in [Halíř97a] for an estimation of profiles of pottery fragments. Because in our case only an approximation of the axis is known from the previous estimation steps, the original method cannot be used directly. Instead, the axis of rotation is improved by the following iterative refinement process depicted in Fig. 3:

1. get an initial position of the axis of rotation a by the optimization approach described in the previous sections
2. generate a set of planes p_i perpendicular to the axis a
3. for every plane p_i:
   (a) detect an intersection between the plane p_i and the fragment F
   (b) fit the detected intersections by a circle c_i
   (c) estimate a center C_i of the fitted circle c_i
4. estimate a new position of the axis $\tilde{a}$ as the line which approximates the centers c_i
5. convergence check: if $\|\tilde{a} - a\| > \varepsilon$, set $a = \tilde{a}$ and go back to the step 2

The algorithm assumes that the detected intersections are circular arcs. With respect to the fact that the position of the axis of rotation can be inaccurate in the beginning, such assumption can be wrong. Thus the algorithm has to cope with distorted intersections and their influence on the circle and line fitting phases (steps 3b and 4). Because of speed and accuracy, both circles and lines are estimated by least squares approaches as proposed in [Thoma89] and [Melen94]. Robustness of these fittings is achieved by an application of M-estimators in the same straightforward way as was used for the robust estimation of the axis of rotation described in the previous section.

PRACTICAL REALIZATION

The proposed estimation method is based on position-independent geometrical properties of pottery fragments. Only coordinates of surface points of the investigated fragment have to be provided for the computation, no other a priori knowledge (such as material characteristics, shape description, color and texture classification etc.) is required. The coordinates can be simply acquired by a range finder, thus the position of the rotation axis can be determined in a fully automatic process without need of experienced archaeologists.

The complete estimation process consists of the following steps (see Fig. 4):

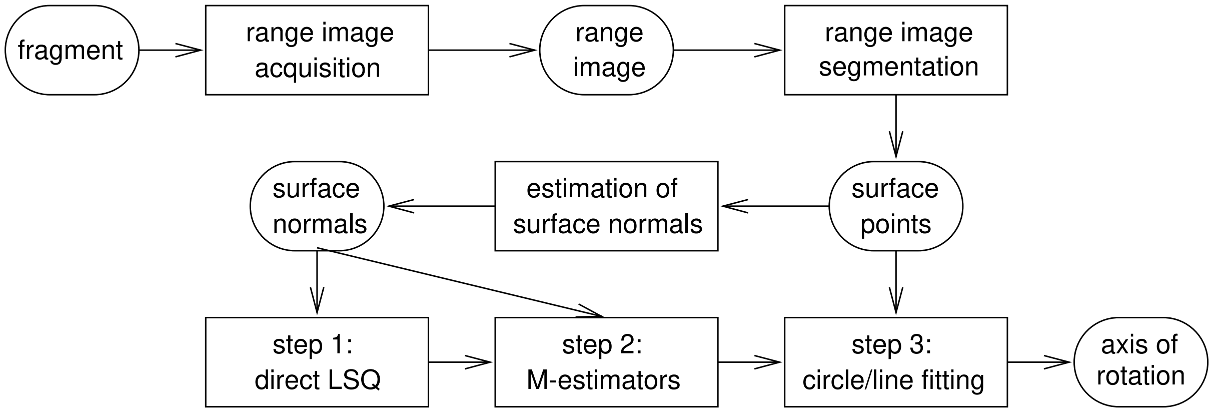


Figure 4: An estimation of the axis of rotation of a pottery fragment

1. *a range image acquisition*: Pottery fragments are scanned by a range finder. A simple range finder created from a laser and a CCD camera connected to a PC-compatible computer [Pajdl95] was used in our experiments.
2. *a range image segmentation*: The acquired range image is segmented and coordinates of the points lying on the surface of the fragment are extracted. There exists a wide range of segmentation algorithms, see [Fitzg96] for an overview.
3. *an estimation of surface normals*: The first two estimation steps are based on surface normals of the fragment. If the normals are not provided directly by the segmentation, they are determined by a plane fitting approach as proposed in [Yacou97].
4. an estimation of an initial position the rotation axis by a *direct least squares optimization*
5. a robust estimation based on *M-estimators*
6. an *iterative refinement* of the estimated rotation axis by a *robust circle and line fitting*

The last three steps are realized exactly as proposed in the previous sections. The whole estimation is implemented in the numerical and visualization system MATLAB [Mathw84].

EXPERIMENTAL RESULTS

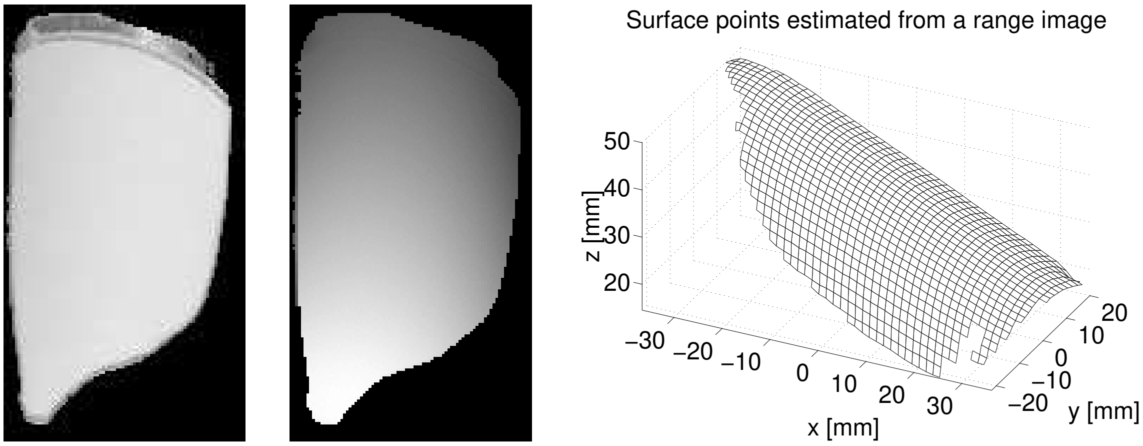
The proposed multi-step model-based approach has been evaluated in many experiments. A set of synthetic range images of objects with known rotation axes was used for an evaluation of accuracy and robustness of the method. We investigated various optimization approaches for least squares minimization, weight functions used in M-estimators, convergence criteria etc.

From the experiments follows that a good approximation of the axis of rotation can be obtained even by the direct least squares minimization. This step is very fast and it converges in 10–20 steps (for the Gauss-Newton optimization method). If there are gross outliers among surface points (typically after bad segmentation of the range image), the robust estimation by M-estimators should be used. The estimation is implemented as a re-weighted iterative process and it requires about 10 steps. From the wide range of weight functions, the fastest convergence was obtained by so called hard redecessors such as Tukey's biweight, Andrew's wave and Welsch's function.

If a precise estimate of the rotation axis is required and the appropriate surface normals are not known in advance, the iterative refinement process should be finally applied. This step is computationally very intensive (detection of intersections, robust circle and line fittings, etc.), but it provides accurate and robust estimates even from small and distorted data sets. About 5 iterations of the algorithm are needed for convergence.

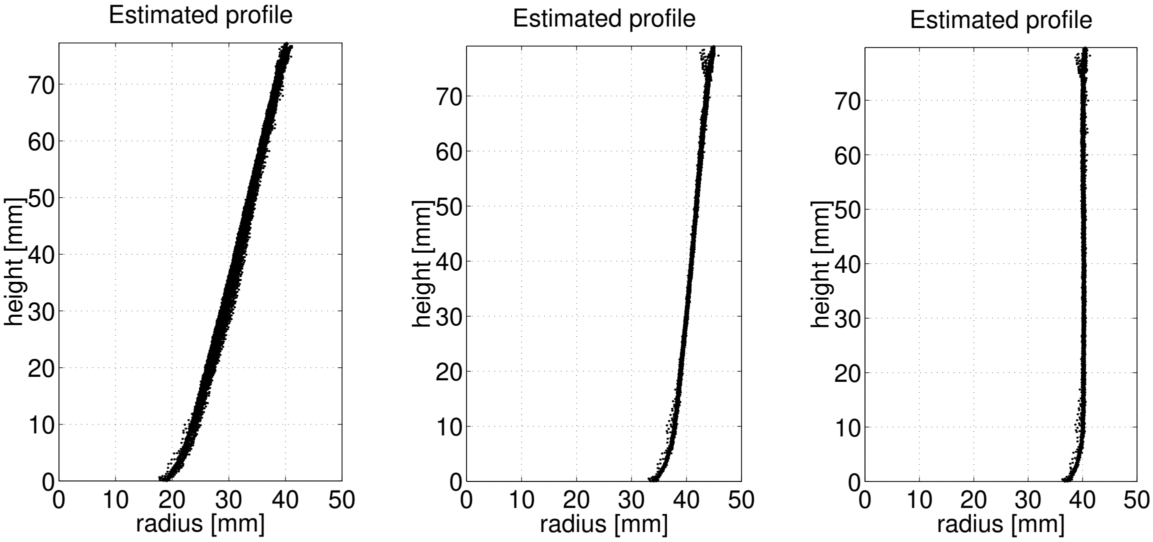
The method has also been tested on real pottery fragments. Because rotation axes of the investigated fragments were not a priori known, correctness of the provided solutions was verified as follows: for the particular fragment, its profile was constructed with respect to the estimated rotation axis as proposed in [Halíř97a] and compared with the profile estimated manually by archaeologists.

Fig. 5 depicts one pottery fragment which was used in the experiments. The fragment comes from a nearly cylindrical-shaped mug with a diameter of 80 millimeters. The fragment was processed as proposed in the previous section, some statistics on the computation is summarized in Table 1. To demonstrate the progress of the estimation, an appropriate profile of the fragment was constructed after each estimation step. The profiles are shown in Fig. 6. It can be seen that for this particular fragment, all three estimation steps had to be used for an accurate estimation. For fragments with more complicated profiles the method converges much faster.



(a) (b) (c)

Figure 5: A real pottery fragment used for the evaluation of the proposed estimation method; a) an intensity image of the fragment; b) an appropriate range image; c) a model of the fragment constructed from the range image



(a) (b) (c)

Figure 6: A profile of the fragment estimated by the proposed method; a) an initial estimate done by the direct least squares optimization; b) a robust estimate done by the M-estimators; c) a final estimate after the iterative refinement step

Estimation step:	I. direct LSQ	II. M-estimators	III. circle/line fitting
Data size:	1002 normals	1002 normals	9062 points
Other info:	Gauss-Newton minimization	Tukey's weight function	30 planes in every step
Iterations:	15	9	3
Time:	2 sec	12 sec	35 sec
Error:	$15°$	$5°$	$<1°$
Note:	bad result due to outliers	better accuracy	very good result

Table 1: Computational statistics of the individual estimation steps. Time was measured in MATLAB v5.2 running on PC-486/66 under Linux. Error is given as the angle difference between the ideal position of the axis of rotation and its particular estimate.

CONCLUSION

This paper is devoted to the estimation of the axis of rotation of radially symmetric objects from their fragments. In particular, fragments of archaeological pottery are considered. Geometrical properties of the fragments are investigated first. It is shown that there exist two positionally independent properties of surface points and normals which are suitable for the estimation of the axis. These properties are described by an appropriate model of a fragment.

Based on the model, three different estimation methods are proposed. An initial position of the axis is determined by a least squares based optimization of a special objective function of surface normals. The computation is fast, but sensitive to outliers in the data. Regarding that, a robust modification of the original method is developed by an application of so called M-estimators. Because the surface normals are typically not a priori known, an alternative method which exploits only coordinates of the surface points is also proposed. The iterative fitting process used for the estimation is slow and computationally intensive, but it produces the most accurate results.

For the practical realization, the individual estimation methods are combined together. The final multi-step approach requires only coordinates of the surface points for the computation. The position of the rotation axis is determined in a fully automatic process, no expert knowledge on the investigated fragments is needed. Regarding that, such automatic estimation can significantly reduce a great deal of routine work of archaeologists during the classification of pottery fragments.

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