Scene Complexity for Rendering in Flatland

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Abstract *The paper studies two problems with the complexity of the polygonal scene for rendering. First it shows that any precise measure for scene complexity must have some special - and maybe surprising - sensitivity. Second it compares analytical and computed solutions of radiosity equation in some cases in flatland.*

1 Introduction

"What are the key issues that are still unanswered or not answered well in realistic image synthesis?", [Shir94]. One of the answers was: "dealing with environmental complexity and with human perceptual issues". This distinguishes geometric and perceptual complexity of the scene, as discussed in [ChSh94]. Our research focused to the geometric complexity of the polygonal scene with N patches.

The inventor of the scene is unknown, however, some reference can be found in [Fole90]. The scene complexity is simply identified with N in many papers. This is due to the fact that many algorithms compute entities, which number depends directly on the number of patches, e.g. the hemicube radiosity algorithm [CoGr85], or the progressive refinement algorithm [Cohe88]. "The relative efficiency of the global Monte Carlo technique - for computing all formfactors - relative to the local Monte Carlo one is equal to the average number **n** of intersections a random line has with the scene", [Sber96]. "Then, if **n** represents roughly the complexity of the scene, the higher the complexity, the higher the relative efficiency".

It seems that both number of patches and **n** are different approaches to develop a suitable geometrical complexity measure. This could be the necessary framework for comparison of different meshing strategies. Our new idea was to identify the scenes where the analytical solution is known and compare it with experiments. This reduced our efforts to flatland. Moreover, we have studied some cases which shed light to the notion of scene complexity.

The paper is organized as follows. Section 2 introduces a special case of scene for which we are able to formulate the exact analytic solution. Section 3 gives the computed results for the same case. Section 4 explains the results obtained for study of scene complexity. Section 5 describes the confrontation of analytic and computed resluts. Section 6 contains the discussion, and conclusion.

2 Analytical Solution of Rendering Equation in Flatland

We have tried to derive the explicite analytical form of the solution in planar case for a scene in the shape of a unit circle. We assumed the reflection coefficient constant along the circle and equal to R; $0 \le R < 1$. Certain fixed part of the circle was the light source with constant intensity L_e which was distributed uniformly to all directions. Our goal was to compute what form will have the function describing global illumination (including the reflected light).

The rendering equation describing in general the planar situation of global illumination of any closed scene with given fixed (but arbitrarily located) placement of light sources is as follows:

$$L_s(x,\vec{w}) = L_e(x,\vec{w}) + \frac{R(x)}{2}\int_X g(x,x_2)\cdot L_s(x_2,\vec{w}_2)\cdot\left(-\vec{w}_2\cdot\vec{n}\right)\cdot\frac{\left(\vec{w}_2\cdot\vec{n}_2\right)}{\left\|x-x_2\right\|}dA(x_2), \qquad (1)$$

where we integrate through all the border of the closed scene; $R(x)$ is coefficient of light reflection in the point x; $g(x,x_2)$ is in general the visibility function which is equal to 1 if the point x is visible from the point x_2 and zero if not; n resp. n_2 are internal normals to the curve creating the border of the scene in points x resp. x_2; w_2 is a unit vector in the direction of a line through points x_2 to x ; and finally $dA(x_2)$ is an element of length measure in the point x_2 .

We have assumed a special case of a circle shaped scene with unit radius and constant reflectivity ($R(x) \equiv R$) and with uniform radiation of all light sources to all directions ($L_e(x,w) \equiv L_e(x)$). This implies a uniform total radiance to all directions (this follows from the equation (1)) ($L_s(x,w) \equiv L_s(x)$). If we parameterize the border of the circle: $x = \cos(2\alpha)$, $y = \sin(2\alpha)$; where α will run throught the interval $\langle 0,\pi\rangle$, then we obtain in the equation (1) the following new form of expressions in the new variables:

Let $x = (\cos(2\alpha) , \sin(2\alpha))$; $x_2 = (\cos(2\alpha_2) , \sin(2\alpha_2))$; then
$w_2 = (\cos(2\alpha) - \cos(2\alpha_2) , \sin(2\alpha) - \sin(2\alpha_2)) \cdot (4\sin^2(\alpha - \alpha_2))^{-1/2}$;
$n = (-\cos(2\alpha) , -\sin(2\alpha))$; $n_2 = (-\cos(2\alpha_2) , -\sin(2\alpha_2))$
$(-w_2 \cdot n) = |\sin(\alpha - \alpha_2)|$; $(w_2 \cdot n_2) = |\sin(\alpha - \alpha_2)|$;
$\| x - x_2\| = 2\cdot|\sin(\alpha - \alpha_2)|$; $dA(x_2) = 2d\alpha_2$.

The new variables (halves of angles) change the general equation (1) to the form:

$$L_s(\alpha) = L_e(\alpha) + \frac{R}{2}\cdot\int_0^{\pi} L_s(\alpha_2)\cdot\left|\sin(\alpha-\alpha_2)\right|d\alpha_2 , \qquad \text{for } \alpha\in\langle 0,\pi\rangle . \qquad (2)$$

If we use the new function $L_o(\cdot) = L_s(\cdot) - L_e(\cdot)$, which represents only the reflected part of the radiance, then the equation (2) has a new form:

$$L_o(\alpha) = \frac{R}{2}\int_0^{\pi} L_o(\alpha_2)\cdot\left|\sin(\alpha-\alpha_2)\right|d\alpha_2 + \frac{R}{2}\int_0^{\pi} L_e(\alpha_2)\cdot\left|\sin(\alpha-\alpha_2)\right|d\alpha_2 . \qquad (3)$$

We can see that this integral equation we can write in an operator formulation

$$I(L_o) = K(L_o) + N(L_e), \qquad (3a)$$

where the functions $L_e(\cdot)$ a $L_o(\cdot)$ are from normed linear space of continuous functions at a cyclic interval $\langle 0,\pi\rangle$, where we can introduce a classic supremum norm. Because it is easy to see that $\|K\| \le R < 1$, then from the method of a geometric series it follows the existence of a linear continuous operator $(I - K)^{-1}$. Thus the equation (3a) can be written as

$$L_o = (I - K)^{-1} (N (L_e)) . \qquad (3b)$$

This assigns to given light sources (to the function L_e) the solution of reflected light (function L_o which we are searching for).

Denote by $L_{e,y}(\cdot)$ the Dirac function with the infinity in the point $\alpha = y$, and zero everywhere else with total value 1 after integrating through the interval $\langle 0,\pi\rangle$. The linear integral operator N allows the convergence in its argument (in the space of distributions) to this Dirac function $L_{e,y}$ using a series of continuous functions L_n ($n=1,2, \ldots$), while the values $N(L_n)$ will converge (in the space of continuous functions) to the limit value $N(L_{e,y})$. Therefore if we $L_{o,y}$ define accordingly to equation (3b), where we replace L_e (using the idea above) by the Dirac function $L_{e,y}(\cdot)$; then

$$L_{o,y} = (I - K)^{-1} (N (L_{e,y})). \qquad (3c)$$

This gives a possibility for a not very extremely difficult proof that the solution of the equation (3b) (function L_o) we can express for a given per partes continuous function L_e as the following integral superposition of base solutions $L_{o,y}$ coming from the equation (3c)

$$L_o(x) = \int_0^{\pi} L_e(y)\cdot(L_{o,y})(x)\cdot dy \qquad (A)$$

Now we need to find the form of those base functions $L_{o,y}$. Without the loss of generality it suffices to find $L_{o,0}$ only. If we denote for short $L_{o,0}(\alpha)$ as $f(\alpha)$, then if we rewrite (3c) back to analogic form (3), then we obtain

$$f(\alpha) = \frac{R}{2}\int_0^{\alpha} f(\alpha_2)\cdot\sin(\alpha-\alpha_2)d\alpha_2 + \frac{R}{2}\int_{\alpha}^{\pi} f(\alpha_2)\cdot\sin(\alpha_2-\alpha)d\alpha_2 + \frac{R}{2}\cdot\sin(\alpha)$$

(4)

Deriving this twice we obtain that $f''(\alpha) + f(\alpha) = R \cdot f(\alpha)$ for $\alpha \in (0, \pi)$. From this follows that $f(\alpha) = c_1\cdot\cos((1-R)^{1/2} \cdot (\alpha - \pi/2)) + c_2\cdot\sin((1-R)^{1/2} \cdot (\alpha - \pi/2))$, where c_1 a c_2 are some constants. From the symmetry of the problem with respect to the transformation $\alpha' = \pi - \alpha$ it follows analogically the symmetry of solution and therefore $c_2 = 0$.

It remains to compute the value of the constant c_1. If we use the identified form of the solution $f(\cdot)$ in the equation (4) and we set $\alpha=\pi/2$ then after integrating both definite integrals we have the following equation for c_1

$$c_1 = c_1\cdot(1 - \sin((\pi/2) \cdot (1-R)^{1/2})\cdot(1-R)^{1/2}) + R/2,$$

what gives c_1 using simple calculation. Finally, the resulting form of the function $f(\cdot)$ is as follows:

$$f(\alpha) = \frac{R}{2\sqrt{1-R}\,\sin(\sqrt{1-R}\cdot\frac{\pi}{2})}\cdot\cos(\sqrt{1-R}\cdot(\alpha-\frac{\pi}{2})) \qquad \text{for } \alpha\in\langle 0,\pi\rangle \qquad (5)$$

We have extended the definition of the function $f(\cdot)$ for all real numbers with the period π, what provides the function f_0 with extended definition. Evidently, $L_{o,y}(\alpha) = f_0(\alpha - y)$ for $\alpha,y \in\langle 0,\pi\rangle$.

Finally, we can now find the explicite solution for some form of function of density of light sources $L_e(\cdot)$. Let $L_e(\alpha) = L_e$ for $\alpha\in\langle 0,\alpha_0\rangle$ and $L_e(\alpha) = 0$ for $\alpha\in(\alpha_0,\pi\rangle$. Then $L_o(\cdot)$ is possible to express using the superposition formula (A). Using both (A) and definition of $L_e(\cdot)$ we obtain the following formulae

$$L_o(\alpha) = L_e\cdot\int_0^{\alpha} f(\alpha-\alpha_2)d\alpha_2 + L_e\cdot\int_{\alpha}^{\alpha_0} f(\alpha+\pi-\alpha_2)d\alpha_2 \qquad \text{for } \alpha\in\langle 0,\alpha_0\rangle,$$

$$L_o(\alpha) = L_e\cdot\int_0^{\alpha_0} f(\alpha-\alpha_2)d\alpha_2 \qquad \text{for } \alpha\in\langle \alpha_0,\pi\rangle.$$

After expressing f using (5) and computing the definite integrals we have obtained after some steps the resulting relationships for the solution $L_s()$ corresponding to our choice of a special function $L_e()$:

$$L_s(\alpha) = L_e + \frac{R\cdot L_e}{(1-R)\cdot\sin(\sqrt{1-R}\cdot\frac{\pi}{2})}\left[\sin(\sqrt{1-R}\cdot\frac{\pi}{2}) - \sin(\sqrt{1-R}\cdot(\frac{\pi-\alpha_0}{2}))\cdot\cos(\sqrt{1-R}\cdot(\alpha-\frac{\alpha_0}{2}))\right]$$

for α such, that $0 \le \alpha \le \alpha_0 \le \pi$, or in other terms

$$L_s(\alpha) = \frac{R\cdot L_e}{(1-R)\cdot\sin(\sqrt{1-R}\cdot\frac{\pi}{2})}\left[\sin(\sqrt{1-R}\cdot\frac{\alpha_0}{2})\cdot\cos(\sqrt{1-R}\cdot(\frac{\pi}{2}-(\alpha-\frac{\alpha_0}{2}))\right],$$

for α such, that $0 \le \alpha_0 < \alpha < \pi$.

When we studied the analogic 3D case - a ball - using the analogic ideas we needed another formulation of the basic equation (1). It has in the space the following form:

$$L_s(x,\vec{w}) = L_e(x,\vec{w}) + \frac{R(x)}{\pi}\int_X g(x,x_2)\cdot L_s(x_2,\vec{w}_2)\cdot(-\vec{w}_2\cdot\vec{n})\cdot\frac{(\vec{w}_2\cdot\vec{n}_2)}{\|x-x_2\|^2}dA(x_2)\,. \qquad (1b)$$

With the same assumptions (constant function $R(x)$ and $L_e(x,w)$ independent from w) we succeeded in a surprisingly simple result for any continuous function (distribution of the density of light sources at the ball surface) $L_e(x)$

$$L_s(x) = L_e(x) + \frac{R}{4\pi\cdot(1-R)}\cdot\int_X L_e(y)dy$$

The comparison of this theoretical result with the results of practical computations for 3D scene we plan to study in the future.

3 Computed Results for Special Test Scenes

We have used an improved version of software by Aleksander Vesel from the University of Maribor, [Vese93]. It is an implementation of a special kind of Monte Carlo radiosity technique, using so-called extended rays. Another flatland radiosity research can be found in [Orti96]. We have created a set of flatland scenes with increasing scene complexities. The objects in this environment are composed from a set of line segments. The light source is typically one (bold) side of the scene. The final light distribution is displayed by a curve along the target line segment (face of an object), see the figures below. The crossing of two resulting curves should be understood as overlapping of two independent visualizations.

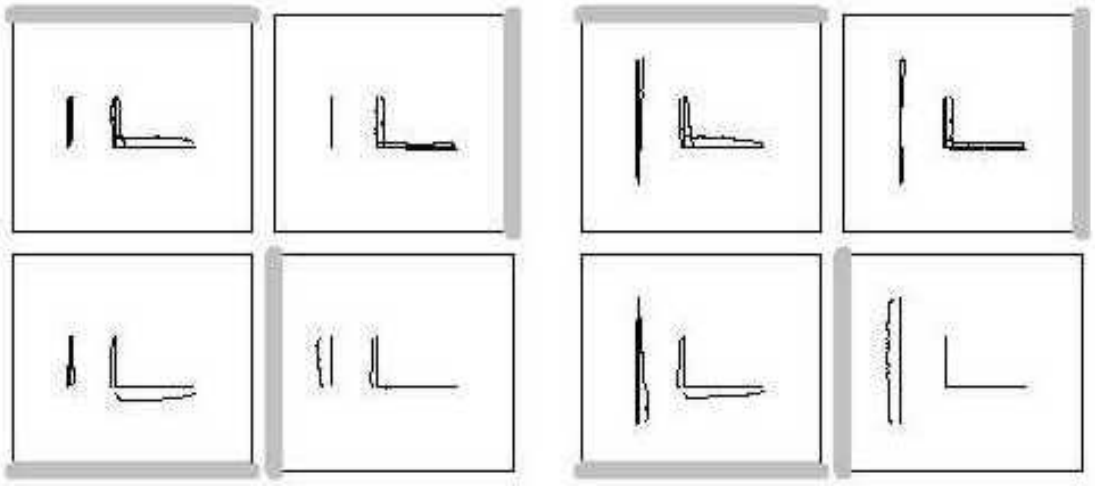
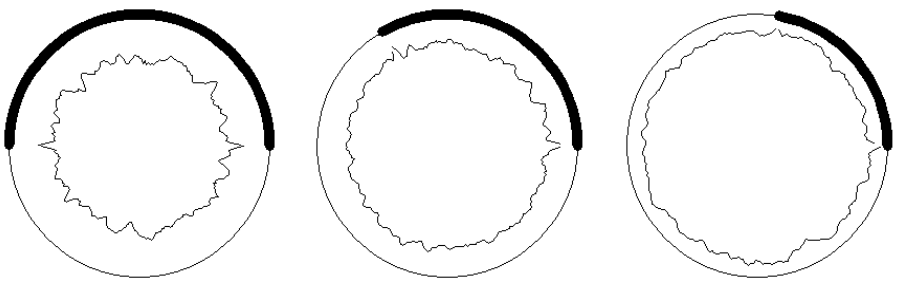
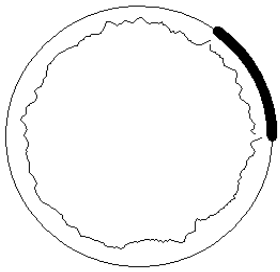


Fig 1: for illustration: two test scenes results for light sources from all sides.

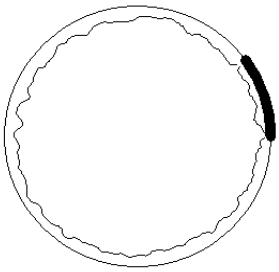
The circle shaped scenes in the following have the bold part as a light source. The rest part completes the empty scene. The zigzag appearance of visualized radiosity is caused mainly by the approximation of the circle by line segments. The technical data for the circle shaped scene are Radius = 500, Total Light Energy = Amount of Patches * Energy Unit, Reflection Factor = 0.9. The table shows the results, where the maximum variance is the maximum of the difference between the radiosity of the patch and the average radiosity in the scene. Angle is the size of the light source.



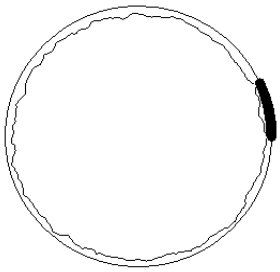
Scene 1. *Scene 2.* *Scene 3.*



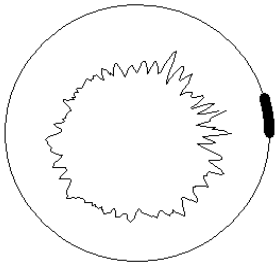
Scene 4.



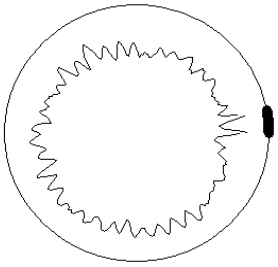
Scene 5.



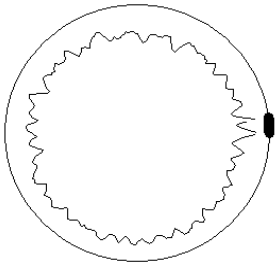
Scene 6.



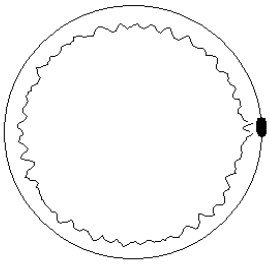
Scene 7.



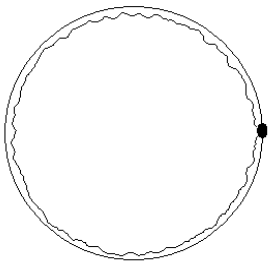
Scene 8.



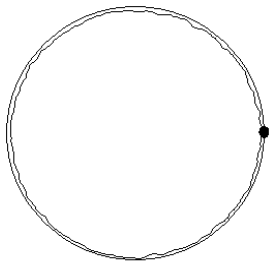
Scene 9.



Scene 10.



Scene 11.



Scene 12.

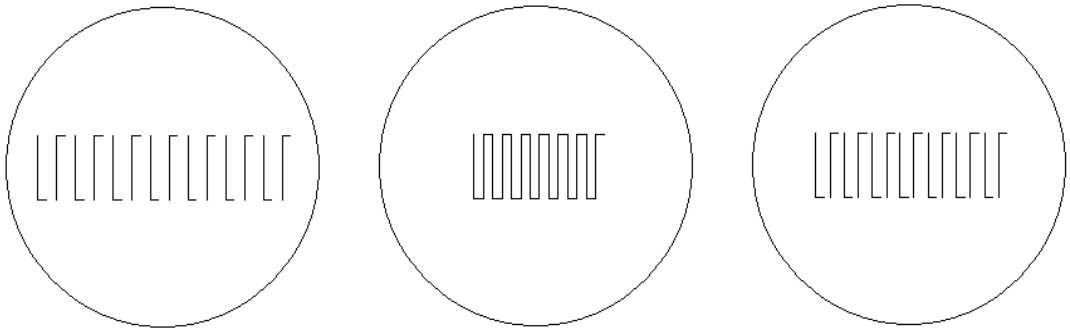
		Number Of Patches	Average Radiosity per Patch	Max Variance	
Scene 1					
Angle =	180	158	268.367	73.117	*Light Source*
Energy Unit =	500	158	284.463	64.648	*Rest Part*
Scene 2					
Angle =	120	105	187.800	44.262	*Light Source*
Energy Unit =	500	210	187.663	49.145	*Rest Part*
Scene 3					
Angle =	80	70	117.633	39.296	*Light Source*
Energy Unit =	500	245	105.813	61.203	*Rest Part*
Scene 4					
Angle =	53.3	47	126.048	32.024	*Light Source*
Energy Unit =	1000	268	126.958	67.303	*Rest Part*
Scene 5					
Angle =	35.6	32	76.100	31.386	*Light Source*
Energy Unit =	1000	284	80.818	38.382	*Rest Part*
Scene 6					
Angle =	23.7	21	53.042	19.572	*Light Source*
Energy Unit =	1000	294	51.158	29.947	*Rest Part*

Scene 7					
Angle =	15.8	14	376.056	81.688	*Light Source*
Energy Unit =	10000	301	334.573	131.470	*Rest Part*
Scene 8					
Angle =	10.5	10	263.458	62.734	*Light Source*
Energy Unit =	10000	305	237.209	101.015	*Rest Part*
Scene 9					
Angle =	7	7	158.089	76.200	*Light Source*
Energy Unit =	10000	309	163.380	89.728	*Rest Part*
Scene 10					
Angle =	4.7	5	95.022	35.398	*Light Source*
Energy Unit =	10000	311	114.093	53.324	*Rest Part*
Scene 11					
Angle =	2.1	2	25.133	1.338	*Light Source*
Energy Unit =	10000	313	45.941	27.542	*Rest Part*
Scene 12					
Angle =	0.6	1	1.426	0.000	*Light Source*
Energy Unit =	10000	314	22.706	14.292	*Rest Part*

It can be observed that the computed results verify the analytic solution (constant radiosity).

3 Another Scene Complexity Experiments

Monte Carlo is one of the methods used for computing the radiosity distribution. As we have mentioned there is an idea of a rough scene complexity measure **n** (denoted there as n_{int}) in [Sber96]. It seems that this measure is not sensitive enough to the scene layout. This is shown by three 2D scenes with the same total length of line segments. We have compared the time needed for computation radiosity distribution in the scenes. Tests giving our observation are presented below. The second scene has the smallest computing time.



1st Scene. *2nd Scene.* *3rd Scene.*

We have used Monte Carlo method for obtaining radiosity distribution. We have also situated all the tested objects into a triangle and rectangle shaped border of the scene. The light source is one third of the border for circle and triangle shaped scene, and one fourth for the rectangle shaped one. For each scene the radiosity distribution was calculated 100 times and we make averages of times and amount of rays needed for computation. We introduce two types of criteria for comparing the scenes. First one evaluate time needed for solving 1 ray (TPR - time per ray), second one is amount of rays computed in 1 second (RPS - rays per 1 second). Both criteria are equivalent. Results are summarized in the table bellow.

Circle

Scene	*Time(milisec)*	*Rays*	*Miliseconds for 1 ray*	*Variance*	*Rays in 1 second*
1	72196.22	6606.64	**10.92782716**	*1.32806*	**91.50950008**
2	67859.44	6527.53	**10.39588328**	*1.26375*	**96.1919226**
3	72823.02	6584.35	**11.06001655**	*1.36237*	**90.41577787**

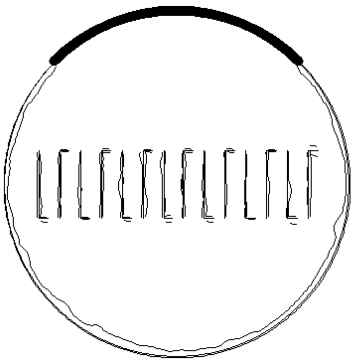
Rectangle

Scene	*Time(millisec.)*	*Rays*	*Milliseconds for 1 ray*	*Variance*	*Rays in 1 second*
1	60808.521	9709.487	**6.262794166**	*0.00984*	**159.6731385**
2	55293.3866	9605.185	**5.756618668**	*0.01507*	**173.7130871**
3	60089.437	9583.765	**6.269919893**	*0.0122*	**159.4916709**

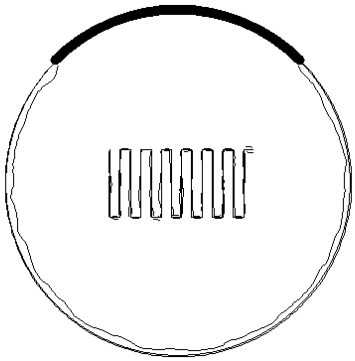
Triangle

Scene	*Time(millisec.)*	*Rays*	*Milliseconds for 1 ray*	*Variance*	*Rays in 1 second*
1	96686.4757	15469.5	**6.250133837**	*0.00862*	**159.9965739**
2	89949.699	15429.11	**5.82987079**	*0.01155*	**171.5303882**
3	98464.1068	15791.2	**6.235376829**	*0.00681*	**160.3752311**

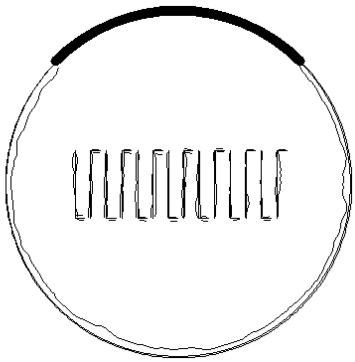
We can conclude that it is shown the difference, which is more than 6% RPS (TPR), between the 2nd and 3rd scene. Therefore, the less complex scene is the 2nd one. On the other hand, the most complex scene is 3rd one. There is a slight distinction between 1st and 3rd one. The rendered scenes with visualized radiosities are presented here:



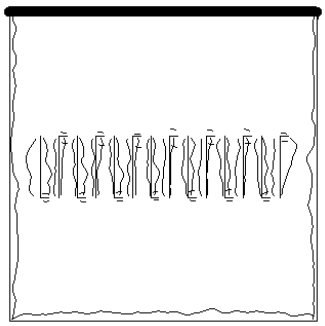
1st Scene.



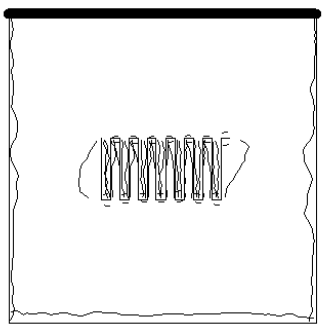
2nd Scene.



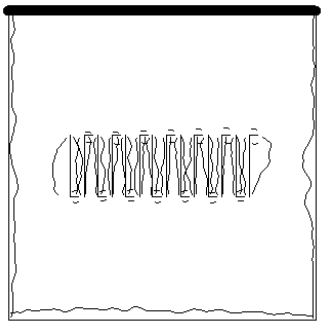
3rd Scene.



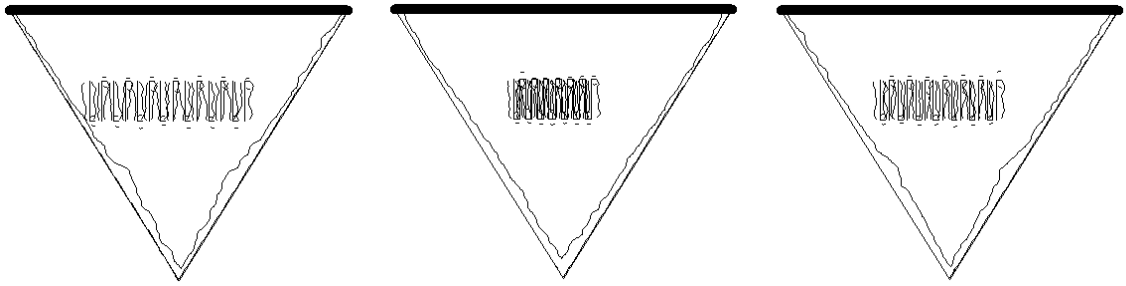
1st Scene.



2nd Scene.



3rd Scene.



1st Scene. *2nd Scene.* *3rd Scene.*

4 Conclusion

We have calculated the complexity for a set of very special scenes in flatland. The results show both that the Monte Carlo method is a good approximation of known exact solution and that the scene complexity is not adequately understood. In our opinion the research of the geometric complexity can improve the understanding of rendering process. Our further research will continue both in analytical and practical efforts, both in 2D and 3D.

We would like to thank anmonymous referees for valuable suggestions.

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