VISIBILITY IN DIGITAL TERRAIN MAPS: A FUZZY APPROACH

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Abstract

The construction of intervisibility models for a terrain is a fundamental tool for environmental design and evaluation of the impact of human intervention. Efficient algorithms are known to address this problem directly on a raster Digital Elevation Map (DEM) of the area under investigation. Unfortunately traditional DEM do not take into account the stochastic aspects of the problem. Randomness plays here an important role: several factors like vegetation, height of the observer, presence of unmapped human artifacts etc. cannot be modeled in advance into a traditional DEM model. We propose to use fuzzy terrain models that incorporates uncertainty and unpredictable variability of the landscape together with a suitable definition of visibility to obtain intervisibility models. Examples of the application of the proposed technique are also reported.

Keywords: Digital Terrain Maps, Visibility maps, GIS.

INTRODUCTION

Given a terrain surface and a viewpoint the classical visibility problem is to detect the portion of the terrain that is visible from the view point [Cohen93]. The applications of this problem are manifold: they range from environmental planning, to selection for best location for viewpoints or for reference lights on a coast, to impact analysis of human artifacts on the landscape.

The solutions to the problem depend on the nature of the digital map available. If the terrain is represented as a polyhedral surface over a triangulated domain several solutions are available. A survey of known techniques in this case can be found in [DeFlor94].

The problem is quite different if the terrain is represented with an array that represent the altitude values sampled on a regularly spaced grid. This last case is referred as "raster DEM". It is important to stress out, as in [Cohen95], that this latter case is not the classical problem of rendering a terrain model through a view-port.

Solutions are available also in the case of raster DEM models of the terrain [Cohen95] but they do not take into account the stochastic aspects of the problem. Randomness, on the other hand, plays, for the G.I.S. applications, an important role: several factors like vegetation, height of the observer, presence of unmapped human artifacts etc. cannot be modeled in advance into a traditional DEM model.

This difficulty is reflected in the common practice of professional environmental architects regarding the evaluation of the impact of human artifacts. A cut-clear decision about visible/invisible pair of points in a natural scene can be done with full confidence only for few points on the map: a robust analysis is frequently impossible for lack of certainty and for the low resolution of the available data. Frequently, moreover, the architects express their conclusions in qualitative linguistic terms that can be naturally modeled with a continuous parameter measuring the "degree of visibility" of pairs of points in a natural scene.

A probabilistic model to address these issues has been proposed in [Sans96]. This method improves on previous approaches [Gros84] and works for contour lines maps. Sansoni's approach is to model the random variations of the landscape that are not faithfully represented in the map with a costly Monte Carlo technique or with few ad hoc techniques: it is, indeed, seldom possible to obtain an explicit probability distribution for the random disturbances and for the imprecise values in the available DEM.

As an alternative, in this paper, we propose to use Fuzzy Digital Elevation Maps (FDEM) as a tool to model both the elevation of the terrain and the uncertainty intrinsic in our knowledge of the natural surfaces. While a traditional DEM gives as an estimate of the elevation in every point a rational number; a FDEM describes elevation with fuzzy numbers [Zimm91], [Anil97a], a relatively new soft-computing concept, that naturally embodies the description of uncertainty.

For this new model of a terrain the traditional concept of visibility must be generalized to a "fuzzy" visibility concept: according to this perspective two points are mutually visible with a certain degree. More precisely, the algorithm proposed in this paper, given a point of observation, assigns to each other point in the map a number in [0, 1] with the following meaning: 0 is given to points that are surely invisible from the observer and higher values, up to 1, are assigned to points that are more and more unlikely to be invisible from the observer. The fuzzy approach advocated here, hence, gives a richer information about the intervisibility of points.

To obtain a soft classification of visible/invisible points we modified the scan-line algorithm proposed in [Cohen95] to the case of fuzzy maps. This has lead to an easy to implement, efficient and robust algorithm.

We have tested the method over fuzzy terrain models built using fuzzy B-splines ([Gallo98a], [Anile97a]) but any other technique that leads to a fuzzy description of the terrain can be successfully employed with the proposed algorithm.

The results obtained are demonstrated with some examples relative to the region of Mount Etna in Sicily, and, in a different application context, relatively to the a segment of the Antarctica ocean bottom.

A final caveat: our technique does not address explicitly the visibility reduction due to atmospheric attenuation but these can be added easily to the model where needed reducing the visibility radius.

The rest of this paper is organized as follows.

In section 2 we introduce the concept of FDEM and of fuzzy intervisibility. In section 3 we describe the incremental visibility algorithm that we have adapted from [Cohen95]. Finally section 4 reports about the experimental results obtained and draws some conclusions.

2. FUZZY DIGITAL ELEVATION MAPS

In this section we shortly introduce the concept of Fuzzy Numbers and of Fuzzy Digital Elevation Maps (FDEM). Using these concepts we are able to give a new definition for intervisibility and a characterization of visible pairs of points that is crucial for the algorithm described in section 3.

2.1 Fuzzy numbers

In order to be self-contained in this sub-section we introduce the concept of fuzzy number. For notations and definitions not explicitly reported here the reader is referred to [Zimm91].

A fuzzy number F is a continuous family of non-empty real intervals I_α with index $\alpha \in [0, 1]$, subjected to the condition:

$$I_\alpha \subseteq I_\beta \text{ if and only if } \alpha \geq \beta \qquad (1)$$

The interval I_α is called confidence interval relative to the presumption level α. Fig.1 gives a graphical representation of some fuzzy numbers and of some confidence intervals.

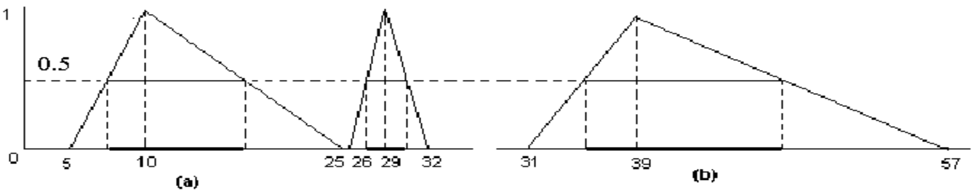

Fig.1 **(a)** Triangular fuzzy numbers with bases [5, 25] and [26, 32] and vertexes 10 and 29; **(b)** the sum of the two fuzzy numbers of Figure 1.(a).

The intended meaning of a fuzzy number generally depends on the applications. Here they are adopted to describe ranges of altitudes embodying uncertainty with the following semantics: a fuzzy altitude, A, describes the relationship between an estimate of the range of possible altitudes in a given point and the amount of pre-assumptions (sometimes called degree of presumption) that leads to that estimate. In other words the parameter α is proportional to the amount of additional information (like statistical evidence, greater control on experimental conditions, subjective preferences, qualitative considerations etc.) available about the terrain measurements. Accordingly, in correspondence of a higher α value uncertainty cannot increase and the range for the possible altitude values may shrink. This is also in agreement with requirement (1). This justifies the use of the expression "presumption level" to indicate the parameter α.

An example can make this concept more clear: suppose to take successive measurements of altitude in a point. If one only keeps the record of minimum and maximum observed values (a minimal amount of additional information or, equivalently, a minimum presumption) the measured data would be correctly represented only with the full range between minimum and maximum. If, on the other hand, an observer keeps a more complete record of the measurements and verifies that some extremal value is observed less and less frequently his presumption that the "real" altitude value is inside a smaller range that excludes such rare extrema becomes stronger. Weighting less, or leaving aside rare or "suspect" values, of course leads to a much better model with regard to precision.

Observe that probability distributions are similar to fuzzy numbers but a fuzzy number in general does not satisfy the probability axioms or vice versa. For this reason fuzzy numbers are sometimes referred as "possibility distributions".

Although a fuzzy number requires an infinite family of confidence intervals, in the application to assure effectiveness, only a finite number of presumption levels are used (typically less than 10). Moreover, all the operations on fuzzy numbers are performed, according to Zadeh's Extension Principle [Zadeh65], independently on each confidence interval. In this way fuzzy arithmetic is a straightforward generalization of interval arithmetic. This point of view is essential in the implementation of such operations [Anile95].

2.2 Fuzzy terrain models

A raster DEM is a rectangular array of numbers a_{ij} that provide an altitude value for the area corresponding to the pixel *(i,j)* in the array.

A raster Interval DEM is a rectangular array of intervals I_{ij} that provide a range of possible altitudes for the area corresponding to the pixel *(i,j)* in the array.

Similarly, a raster FDEM is a rectangular array of fuzzy numbers f_{ij} that provide a fuzzy description of the altitude value for the area corresponding to the pixel *(i,j)* in the array.

A DEM can be seen as a discrete sampling on a lattice from a continuous domain of a 2D surface. Similarly a raster FDEM can be regarded as a discrete sampling on a lattice from a continuous domain of a family of thick layers (parameterized with the presumption levels α).

For each fixed α value, hence, one can suppose a thick layer (interval surface) whose sampling at grid points generates the corresponding levels of the fuzzy numbers f_{ij}. Layers become thinner as the presumption level grows. A visualization of the bounding surfaces of layers relative to different α values is reported in Fig.2. A FDEM can hence be considered as a family of Interval DEM parameterized with presumption levels α.

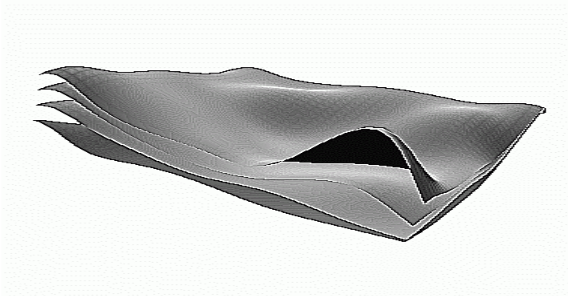

Fig. 2 . A visualization of the bounding surfaces of layers in a fuzzy surface relative to different α values

FDEM can be, in principles, obtained in several ways. A method to obtain a FDM using B-splines, fuzzy arithmetic and a possibilistic analysis of the data is reported in [Anile97a], [Anile97b], [Gallo98a] and [Gallo98b]. These approaches generalize similar techniques using intervals proposed in [Patri95] and [Patri96] and several successive papers of the same MIT research group.

It is important to observe that the visibility analysis proposed below does not depend on the technique used to obtain a FDEM: it only assumes that such a representation for the terrain is available.

2.3 Fuzzy visibility

To discuss visibility, for sake of clarity, we consider a 2D setting as in Fig.3 where the concept of classical intervisibility is illustrated. Points A and B are mutually visible because the line of sight joining them does not meet any obstacle. The points A and C, on the other hand, are mutually invisible because the line of sight joining them "goes through" the terrain. The generalization to 3D is straightforward.

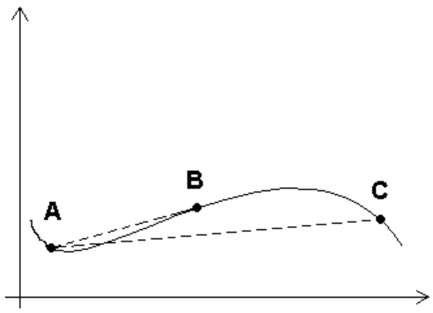

Fig. 3 This diagram illustrates the concept of intervisibility for a classical terrain profile.

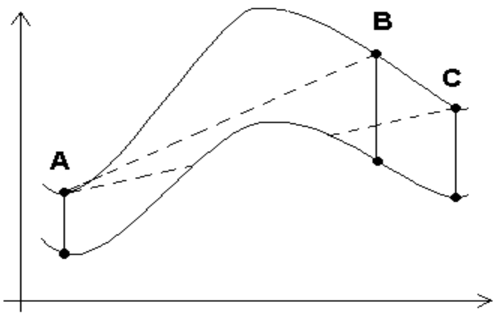

Fig. 4 This diagram illustrates the concept of intervfisibility for an interval terrain profile.

Let us consider, at first, the situation when a terrain is described with intervals as in Fig.4. In this case we adopt the following definition:

Definition 1. A location A and a location B are *possibly mutually visible* in the interval model of the terrain if and only if there exists a line of sight L, from a point in the interval associated with location A to a point in the interval associated with location B, such that L lies all above the lower boundary of the strip describing the terrain.

For example in Fig.4 the pair of locations A and B are possibly mutual visible, while the pair of location A and C are not.

Some observations are useful to convince the reader about the soundness of the previous definition. When a region is described with intervals the only available information is that the real terrain contour lies inside of the strip: nothing can be said about its effective location. For example the strip in Fig.5 allows the contour in dashed lines as well as infinitely many others. Consider the pair of locations A and B. If the real contour of the terrain follows the lower boundary of the strip the two locations would be invisible. But we cannot exclude that the real terrain contour follows the dashed line, and in this case the two locations would be mutually visible. Lacking further information all we *may* say is that the two locations are *perhaps* mutually visible. It is easy to see that the above definition is in complete agreement with this observation.

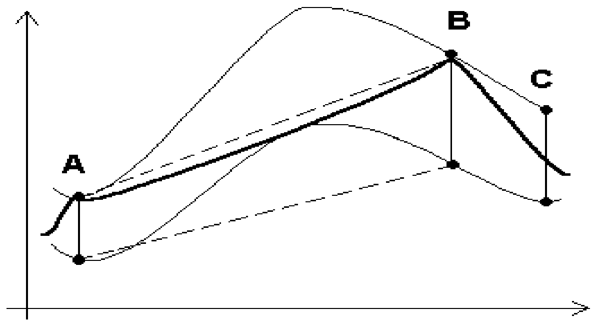

Fig. 5 The real terrain contour is shown inside the interval spline of the possible location with a solid line.

Observe that a stronger statement is made when two points are pronounced *not possibly mutually visible*: in this case it does not matter what the real terrain contour is, the line of sight joining the points will *always* meet an obstacle. This is, for example, the case of locations A and C in Fig.4. In this case too the generalization to surfaces is straightforward.

It is now natural to propose the following definition for fuzzy visibility:

Definition 2. Two locations A and B on a FDEM *are possibly mutually visible with degree* α , if and only if they are possibly mutually visible in the Interval DEM relative to α.

As an example consider the situation in Fig.6: there locations A and B are possibly mutually visible with degree 0, but they are not possibly mutually visible with degree 0.5.

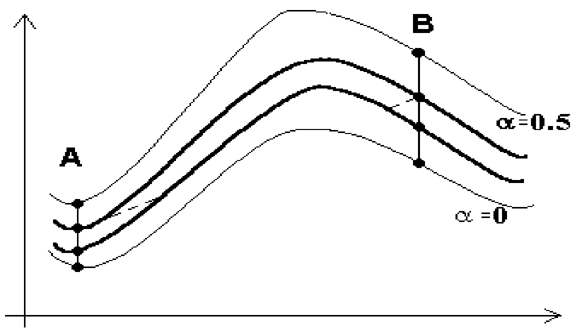

Fig. 6 A and B are possibly mutually visible with degree 0, but they are not possibly mutually visible with degree 0.5.

In correspondence to a fixed value α, given an observation point A, the α-basin of possible visibility of A, $V_{\alpha,A}$, of the map is defined as the set of all the locations in the map that are possible mutually visible with A with degree α. Increasing values of the presumption level α give smaller basins. Three visibility basins of the location A are shown in Fig.7.

The complement of $V_{\alpha,A,}$ $D_{\alpha,A}$, is the set of points that are hidden from A with presumption level α. Increasing values of α give larger $D_{\alpha,A}$. These sets are also called Fuzzy Dead Zones of the terrain relatively to A.

One can also associate to each point P, of the map its Maximum Degree of Visibility from A, $MDV_A(P)$. This number is the higher presumption level α such that $P \in V_{\alpha,A}$. $MDV_A(P)$ is a real number in [0,1] and can be used to associate an intensity level to point P if one wish to represent visually the intervisibility information.

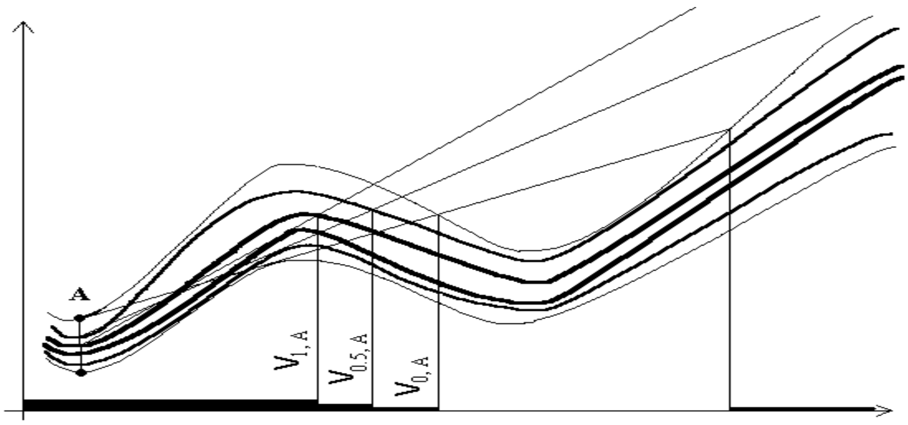

Fig. 7 Three visibility basins of the location A

The generalization of the concept of fuzzy degree of intervisibility to surfaces is straightforward.

Definition 1 is difficult to translate into an effective algorithm because it requires to check infinite lines of vision. A simple observation however reduces the number of required checks to one. Let T_A and T_B be the highest altitudes associated with locations A and B in an Interval DEM. The following Lemma is easily proved:

Lemma 1. Location A and B are possibly mutually visible in an Interval DEM if and only if the line of vision joining T_A and T_B is completely above the lower boundary of the interval surface.

This fact is the base of the algorithm described in the following section.

3. AN ALGORITHM FOR FUZZY VISIBILITY TEST

In this section we describe an algorithm for the classification of the points in a FDEM relatively to their visibility degree from an observation point A. The algorithm presented here has been adapted from the algorithm proposed in [Cohen95] taking into account the new representation of the data.

The common practice of Fuzzy Arithmetic is to work with only a finite number of presumption levels $\alpha_1,\ldots,\alpha_k$. It is clear from Definition 2 in the previous section that the problem of visibility in a FDEM is hence reduced to k visibility problems in k Interval DEM. For this reason, in the following we first present an algorithm for testing intervisibility in the Interval DEM case. An algorithm for the fuzzy case is obtained straightforwardly.

Following [Cohen95] we first consider the 2D situation of Fig.2. The general case of surfaces will be successively considered.

Let $\{u_i\}_{i=0\ldots m}$ be a discretization of the U axis into $m+1$ uniformly spaced points. Let I_i the range of altitudes that the Interval DEM assigns to the point u_i. We denote with P_i the pair (u_i, I_i). We call this pair an *interval location*.

Let $\{T_i\}_{i=0\ldots m}$ be the sequence of the highest altitudes associated with location P_i,'s and let $\{B_i\}_{i=0\ldots m}$ be the sequence of the lowest altitudes associated with location P_i's. The location $P_0 = (u_0, I_0)$ is assumed to be the observer location. The algorithm scans the sequence $\{P_i\}_{i=1\ldots m}$ and for each location determines whether it is visible from P_0 or not.

From Lemma 1 in the previous section to test intervisibility between P_0 and P_j is equivalent to test that the line of sight joining T_0 and T_j does not intersect the contour described with the B_i's.

3.1 The algorithm for the 2D case.

The algorithm proposed here is based on the following simple observation: for each index $i=1,\ldots,m$ let L_i be the line joining T_0 with B_i and let S_i be the upper half-plane determined by L_i Then a point T_h is visible from T_0 if and only if it lies in the region $R_h = S_1 \cap S_2 \cap \ldots \cap S_{h-1}$.

This situation is easily tested ([Prep85]) checking that the slope of the line joining T_0 with T_h is greater than the maximum slope of the lines joining T_0 with the B_i's, $i<h$.

This leads to a procedure that is best described in the following pseudo-code:

```
ClassifyLocations1( )
{
    Set P0 visible;
    largest_slope = - ∞;

    For i =1 to m do
    {
        if slope(T0,Ti) > largest_slope   then set Pi visible
                                          else set Pi invisible;
        if slope(T0,Bi) > largest_slope   then set largest_slope = slope(T0,Bi);
    }
}
```

Since slope computation requires a division this code may turn out inefficient.

It is possible in alternative to compare the slope of the line joining A and B, with the slope of the line joining A and C checking the sign of the cross product $(B-A)\times(C-A)$ ([Prep85]). In this case instead of having a *largest_slope* variable one has only to keep track of the index k such that the line joining T_0 with B_k produces the maximal slope during the scan.

The pseudo-code can hence transformed as follows:

```
ClassifyLocations2( )
{
    Set P0 visible;
    Set P1 visible;
    Set k=1;   /* index of largest slope insofar */

    For i =2 to m do
    {
        if (Bk-T0)(Ti-T0) > 0 then set Pi visible
                             else set Pi invisible;
        if (Bk-T0)(Bi-T0) > 0 then k=i; /*new index for the largest slope */
    }
}
```

A quick review of the code and of the formula for cross product evaluation shows that four products, six sums and two comparisons are required in each execution of the internal loop of the procedure ClassifyLocations2.

Following [Cohen95] a further speed up can be obtained with an incremental approach to the cross product computation, reducing the number of product operations required. The details of this final optimization are not reported here.

3.2 The case of surfaces.

Following the approach of [Cohen95] the 2D solution described in the previous sub-section can be applied to obtain a solution for the case of surfaces embedded in 3D. The idea is to "cut" 1D silhouettes of the surface in 3D along all the directions going out from the observation point. Theoretically there are infinite directions, however, since we are dealing with a raster (discrete) FDEM it is enough to consider only a finite number. Precisely the directions needed are those obtained joining the observation point with the pixels on the boundary of the map as shown in Fig.8. This set of direction is dense, i.e., covers all the pixels in the map: any given pixel belongs to at least one direction in the set. There are, of course also redundancies and pixels may be processed more than once as belonging to more than one line.

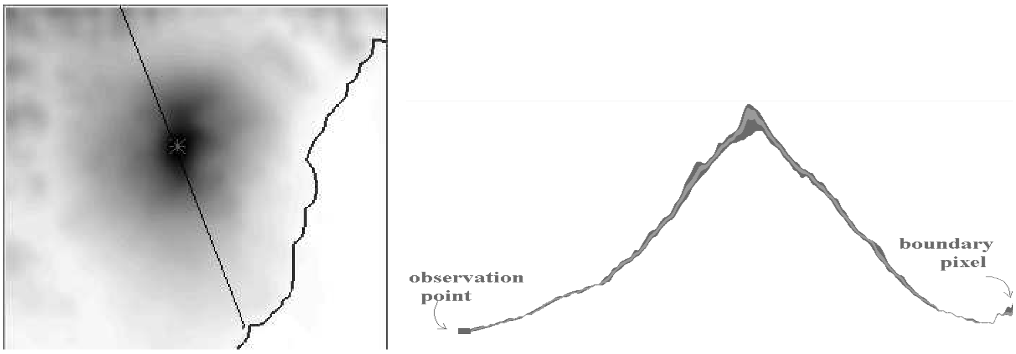

Fig. 8 direction obtained joining the observation point with one pixel on the boundary

The lines along these directions can be computed using any of the well known incremental algorithm for line tracing [Foley90] but the outcome depends on the kind of discrete topology adopted. The lines to extract can be 4-connected or 8-connected. The first choice produces pixel sets that are somehow "thicker" and may lead to "false invisible" locations. Conversely 8-connected lines uses less pixels and may produce "false visible" locations. The second choice however leads to a faster processing: less pixels per section have to be processed and produces less redundancy: a pixel in the map, on the average, is processed fewer times than in the case of 4-connectness. For these reasons 8-connectness has been the choice done in the implementation that we have realized.

The inaccuracies introduced in any of the two cases can be considered as aliasing effects. Better results can be obtained processing more directions. This can be done subdividing the boundary at sub-level pixels. This strategy gives more precise results but has a greater computational cost. Since in a FDEM we are already dealing with an uncertain model this greater computational effort is seldom justified.

In any case every pixel in the map is processed at most a constant number of times, and since each processing step requires a constant number of arithmetic the algorithm is $O(n)$ where n is the total number of pixels in the map or in the area of interest in it. The linear complexity of the algorithm in [Cohen95] for DEM is, at least asymptotically, preserved for the case of Interval DEM and for FDEM.

4. Experimental Results

We have implemented a fully incremental version of the algorithm reported in section 3 and tested it for several terrain. Here we report few visibility maps relative to the geographical area surrounding Mount Etna on the Eastern Coast of Sicily. In this case the fuzzy model of the surface has been obtained from a set of about seventy thousand scattered points using fuzzy B-Splines ([Anile97a],[Anile97b]).

Fig.9 is an elevation map, in gray tones, of the Mount Etna area. For sake of visualization the altitude of lower boundary of the interval level relative to α=0 has been used to construct the map. In the map the black solid line indicates the coast line and the cross indicates the highest point of the volcano. Taormina is a city, high on the coast and it well known for the breathtaking views that offers of the region. Fig.11 shows a fuzzy visibility map when the observation point has been put in Taormina. The map uses white to indicate the points visible at highest degree and a scale of darkening grays to indicate the points with lower degrees of visibility.

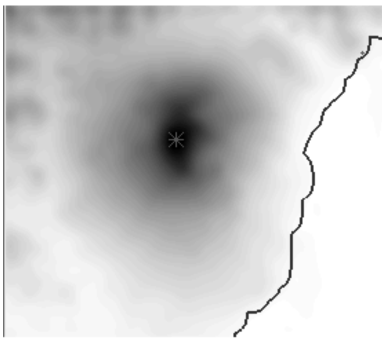

Fig.9 Elevation Map

Fig.10 Classical visibility map

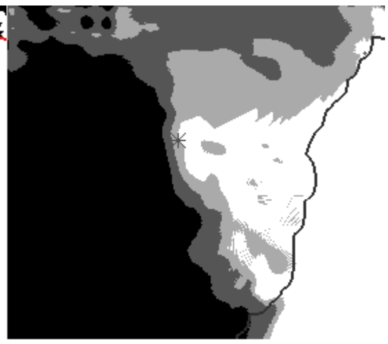

Fig. 11 Fuzzy visibility map

The map is not entirely realistic because of the relatively low resolution adopted and because it has been obtained without taking into account the earth curvature and the atmospheric attenuation. Both factors are indeed important when dealing which such a large area (about 2500 squared Km.). In any case the results are consistent with reality even in several small details. For example the map shows that the top of Etna is visible with highest degree, while the large depression immediately at NE of it (known as Valle del Bove) is visible only at smaller degree. The information provided with a fuzzy visibility map is certainly richer than the information carried by a classical visibility map as a comparison of Fig.11 with Fig.10 makes evident. Fig.12 shows the visibility relative to an observation point located in the tourist harbor of Ognina (Catania suburbia). Finally Fig.13 shows the visibility from a point located 40m higher over the top of the S. Gregorio hill, where the TV repeaters serving the region are located. The map proves the good coverage offered from antennae located in such position.

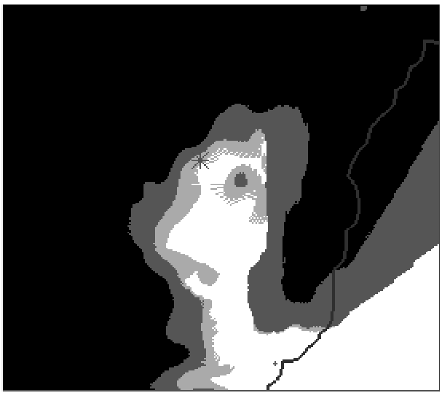

Fig.12 Observation point located in Ognina

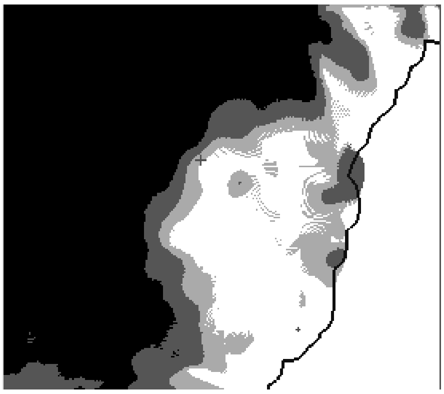

Fig.13 Observation point located in S. Gregorio

5.Conclusions

We have introduced a new graded concept of intervisibility that tries to take into account the uncertain and unmapped features of a terrain. The new concept is sound and leads to an efficient algorithm for visibility map construction. The effectiveness of the algorithm and of the fuzzy visibility concept has been demonstrated with some examples.

Acknowledgments.

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References

[Anile95] M. Anile, S. Deodato, G. Privitera. *Implementing Fuzzy Arithmetic*. Fuzzy Sets and Systems 72 (1995) 239-250.

[Anile97a] M. Anile, B. Falcidieno, G. Gallo, M. Spagnuolo, S. Spinello, *Uncertain data processing with fuzzy B-splines.* Preprint, submitted for publication to Fuzzy Sets and System. 1997.

[Anile97b] M. Anile, B. Falcidieno, G. Gallo, M. Spagnuolo, S. Spinello, *Approximation of bathimetric data with FBS.* (poster) SIAM CAGD 97 Conference, Nashville, TN, 1997.

[Cohen93] D. Cohen and A. Shaked. *Photo-realistic imaging of digital terrains.* Computer Graphics Forum, 12(3):363-373, 1993.

[Cohen95] D. Cohen and A. Shaked. *Visibility and Dead-Zones in Digital Terrain Maps.* Eurographics '95, Volume 14, (1995), Number 3.

[DeFlo94] L. De Floriani and P. Margillo. *Visibility algorithms on triangulated digital terrain models.* International Journal of GIS, 8(1):13-41, 1994.

[Foley90] J. D. Foley, A. VanDam, S. K. Feiner and J. F. Hughes. *Computer Graphics Principles and Practice.* Addison Wesley, Reading (1990).

[Gallo98a] G. Gallo, M. Spagnuolo. *Uncertainty Coding and Controlled Data Reduction using FBS.* Preprint, Catania 1997, submitted to CGI98.

[Gallo98b] G. Gallo, I. Perfilieva, M. Spagnuolo, S. Spinello. *Geographical data reduction using mountain function.* Preprint, Catania 1997.

[Gros84] G. Gros and B. Janin. *L'analyse automatique des composantes visuelles des sites et project. Des questions simples pour un problème complexe.* Revue Gen. Routes Aerodromes, N.613 (1984) pp. 85-93.

[Patri95] N. M. Patrikalakis et Alii. *Virtual enviroment for Ocean Visualization and Exploration.* Proc. Computer Graphics Technology for the exploration of the sea, CES95, Rostock, May1995.

[Patri96] N. M. Patrikalakis at Alii. *Robust Interval Solid Modeling*. Computer-Aided Design, vol. 28, no. 10, pp. 810-817, 1996.

[Prepa85] F. P. Preparata and M. I. Shamos. *Computational Geometry: An Introduction.* Springer-Verlag, New York, 1985.

[Sanso96] C. Sansoni. *Visual Analysis: a new probabilistic technique to determine landscape visibility.* Computer-Aided Design, vol. 28, no. 4, pp. 289-299, 1996.

[Zadeh65] L. A. Zadeh. *Fuzzy Sets.* Information and Controls, 8:338-353, 1965

[Zimme91] H. J. Zimmermann. *Fuzzy Set Theory and its applications*. Kluwer Acad.Pub. 1991.