Application of Coons and Gordon Surface Modeling Apparatus in Reverse Engineering of Turbomachinery Components

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Abstract

The article discusses a pragmatic approach to surface modeling of turbomachinery components using the Koons and Gordon surface apparatus. These surface modeling techniques are used to prepare CAE models from highly detailed and often unusable computational models. A rationale is given for the usefulness of the Koons-Gordon algorithm in the field of turbomachinery, as well as its application in various geometric scenarios.

A positive feature of the algorithm is the possibility of more flexible adjustment of models when new structural patterns are discovered during the development of a patch, having only four curves as input. The result is a 3D mesh generated from a set of surfaces. The motivation for the work was that at the moment there was no open source project capable of building such a mesh from a set of Coons patches. The presented Gordon surface interpolation algorithms are implemented and published in the open source CAD system “Analysis Situs”.

Keywords

Computer-aided design, geometric modeling, reverse engineering, coons surface, gordon surface.

1. Introduction

In the process of working on the "Analysis Situs" project [8], a task was received from the customer related to the construction of a controlled loft. Because we only have a curve network in the input and the user expects a surface in the output, we picked the Coons-Gordon algorithm [1].

This paper addresses two approaches for the Coons-Gordon surface construction: the global one, which considers the network of curves as a whole and then approximates/interpolates this network; and the local one, which considers a set of "sub-rectangles" obtained from the network of curves and then builds a Coons surface [2] on each of them and connects them into a single surface. We discuss the advantages and disadvantages of the global and local approaches and explain why we choose the local method as the best and implement this in "Analysis Situs" project.

2. Coons-Gordon surface

For the first time, the apparatus of boundary curves and conjugation functions, which became the basis for the development of surface modeling, in particular, appears in the monograph by S. Coons in 1967 [2]. There he describes the algorithm for constructing Coons patches.

Let us be given three functions $f(u,v), g(u,v), h(u,v)$ that define the coordinates (x,y,z), respectively, where u, v are independent parameters. Then, if we fix one of the parameters u or v, we can find the values of the points (x,y,z) along one curve. If we fix this parameter with a certain step, then we can get a set of curves that will lie on one surface and thus define it. Afterward this set of curves will define a set of sub-rectangles, each of which is bounded by its four curves $\boldsymbol{c_0}$, $\boldsymbol{c_1}$, $\boldsymbol{b_0}$, $\boldsymbol{b_1}$, where each such a sub-rectangle is called a Coons patch (figure 1, 2).

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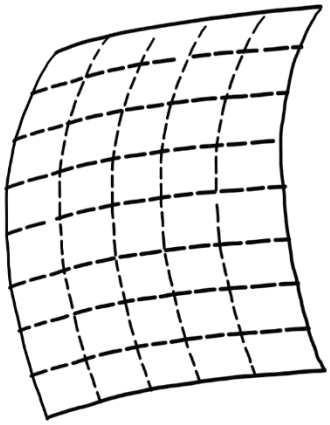

Figure 1: Network of curves for Coons patching

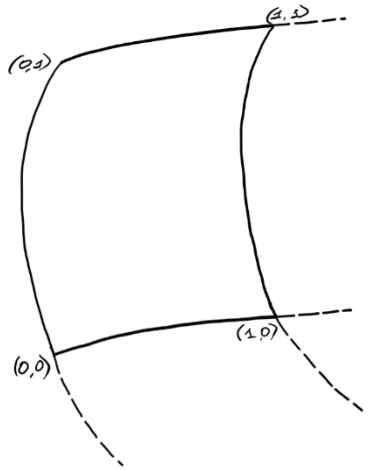

Figure 2: Coons patch

The designed edge curves on each patch can be represented as the isoparametric curves $\boldsymbol{C}(0,v), \boldsymbol{C}(1,v), \boldsymbol{C}(u,0), \boldsymbol{C}(u,1)$, corresponding to the corner intersection points indexed as

$$\left(\boldsymbol{P}_{\{i,j\}}, \boldsymbol{P}_{\{i,j+1\}}\right), \left(\boldsymbol{P}_{\{i+1,j\}}, \boldsymbol{P}_{\{i+1,j+1\}}\right), \left(\boldsymbol{P}_{\{i,j\}}, \boldsymbol{P}_{\{i+1,j\}}\right), \left(\boldsymbol{P}_{\{i,j+1\}}, \boldsymbol{P}_{\{i+1,j+1\}}\right).$$

Then the definition of the Coons patch can be expressed using linear interpolation.

$$\begin{aligned}\boldsymbol{C}(u,v) = {} & \boldsymbol{C}(u,0)(1-v) + \boldsymbol{C}(u,1)v + \boldsymbol{C}(0,v)(1-u) + \boldsymbol{C}(1,v)u - \boldsymbol{C}(1,v)u - \boldsymbol{C}(0,1)(1-u)v \\ & - \boldsymbol{C}(1,0)u(1-v) - \boldsymbol{C}(0,0)(1-u)(1-v) - \boldsymbol{C}(1,1)uv\end{aligned}$$

where $\boldsymbol{C}(u,0)(1 - v\boldsymbol{C}(u,1)v$ – linear interpolation of one set of curves, $\boldsymbol{C}(0,v)(1-u) + \boldsymbol{C}(1,v)u$ – linear interpolation of another set of curves, $\boldsymbol{C}(0,1)(1-u)v + \boldsymbol{C}(1,0)u(1-v) + \boldsymbol{C}(0,0)(1-u)(1-v) + \boldsymbol{C}(1,1)uv$ – bilinear interpolation.

Using this formula we can build a set of Coons surfaces which we can work with further, for example, combine them into one single surface using interpolation or approximation.

Since curves can be of various degrees and curvature, linear interpolation may not work as expected by the user. To solve this problem, William Gordon introduces a concept of projector (approximator or interpolator) of curves from the Φ space into the Γ space.

In 1983, he published his article [1] in which he introduced a Boolean Sum [3]. The Boolean Sum is the combination of projectors P_i and P_j. There, he proposes to use the projector on Coons patches, that is, to approximate or interpolate many surfaces before joining them into a single surface. In the article, he describes two main approaches - global and local.

In the global method, the network of curves is treated as a single entity. $P_u(F) = \sum_{i=1}^{M} \phi_{i(u)} F(u_i, v)$ and $P_v(F) = \sum_{i=1}^{N} \phi_{i(v)} F(v, u_i)$ are considered as projectors, where ϕ is interpolation basis functions (for example, Lagrange interpolation or cubic spline interpolation), u_i, v_i are curves. The Boolean Sum of these projectors is the expected surface.

The advantage of this approach is that we only need two projectors and do not need to build a set of Coons patches, so the result can be achieved by avoiding several steps described in the Coons monograph.

The disadvantage of this approach is the need to approximate/interpolate the network of curves: on objects with a large curvature, the projector will give a new surface that is significantly different from the expected one and approximation of large objects takes a long time. So it is important to have enough curves that have a lot of inner knots to minimize approximation error. It means the accuracy of output model depends on the input network - the locations, degree and number of knots of the curves. These factors are not predictable, we do not know what type of curves we get and we cannot be sure that the result model is valid to produce. If we use Coons patch algorithm on such complex models it can fail because it works only with parametric curves, not b-splines.

The local method includes the Coons patch method. The network of curves is represented as a set of Coons patches. (u, v) the parameters of each patch are limited by the interval (0,1). After all patches

are built, an interpolation function is applied to them, which interpolates only the corners of each patch. Boolean Gordon sum

$$\boldsymbol{P_i} \oplus \boldsymbol{P_j} = (1-u)\boldsymbol{F}\,(0,v) + u\boldsymbol{F}(1,v) + (1-v)\boldsymbol{F}(u,0) + v\boldsymbol{F}(u,1) - (1-u)(1-v)\boldsymbol{F}(0,0) - (1-u)v\boldsymbol{F}\,(0,1) - u\,(1-v)\boldsymbol{F}\,(1,0) - uv\boldsymbol{F}\,(1,1)$$

interpolates patches along the perimeter.

The advantage of this approach is that we process small and simpler curve shapes and build a surface on each of them. Thus, many of these patches define the expected surface as accurately as possible. Since among these patches there will definitely be a pair that has a common curve, such surfaces can be connected without significant loss of accuracy, which means that the result will also describe the expected surface as accurately as possible.

The disadvantage of this approach is the problem that appears during the concatenation of built patches. It is important to choose the correct way to join the set of Coons patches.

3. Implementation of the Coons-Gordon algorithm

The Coons-Gordon method is implemented in many projects, including open source projects, for example, Matlab library NURBS Toolbox [4], TiGL [9], Analysis Situs [8]. However there we want to get NURBS (Non-Unify Rational B-Spline Surface) and it means we have a b-spline curves in the input. To solve this problem the projects use network of curves approximation that we want to avoid (see the example of such algorithm implementation in the work [5]).

Our work is based on the algorithm implemented by TiGL - open-source project, which build Gordon surfaces with B-Splines. TiGL is a software designed as a direct implementation of the data model containing the common parametric aircraft configuration scheme (CPACS) [6].\\

The idea of TiGL algorithm is to build B-Spline curves, interpolate them to increase the knot number, join them to one form using Gaussian.

The authors classify curves on profiles and guides. The network of curves must be closed, relative to the profile and guide curves. In addition, the following condition must be met - all profile curves must intersect with the guide curve at the same curve parameter, and all guide curves must intersect the profile curve at the same parameter.

The Boolean sum formula from the Gordon article is also taken as the basis for the Gordon surface. However, the authors of TiGL use B-splines, so they introduced additional conditions on the curves on which the Gordon surface is built:

- The same degree of curves
- Common hotspot
- Fulfillment of prefiously described condition for profiles and guides

Then the Gordon surface can be presented as

$$s(u,v) = \sum_{i=0}^{n-1}\sum_{j=0}^{N-1} P_{i,j}\boldsymbol{N_i^v}(u,\tau_f)\boldsymbol{N_j^{d_f}}(v,\psi_i) + \sum_{j=0}^{m-1}\sum_{i=0}^{M-1} Q_{i,j}\boldsymbol{N_i^{d_g}}(u,\psi_f)\boldsymbol{N_j^{\mu}}(v,\tau_i) - \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} T_{i,j}\boldsymbol{N_i^{d_g}}(u,\psi_g)\boldsymbol{N_j^{d_f}}(v,\psi_f)$$

where P - control points of profiles, Q - control points of guides.\\

For $P_{i,j}, Q_{i,j}, T_{i,j}$ the following conditions must be met:

$$\sum_{j=0}^{N-1} P_{i,j}\boldsymbol{N_j^{d_f}}(v_k,\psi_f) = p_i^{(k)}, i = 0,\dots,n-1$$

$$\sum_{j=0}^{N-1} Q_{i,j}\boldsymbol{N_i^{d_g}}(u_l,\psi_g) = q_i^{(l)}, j = 0,\dots,m-1$$

$$\sum_{i=0}^{n-1}\sum_{j}^{N-1} T_{i,j}\boldsymbol{N_i^{d_g}}(u_l,\psi_g)\boldsymbol{N_j^{d_f}}(v_k,\psi_g) = \alpha_{l,k}$$

This ensures that the first term interpolates profile curves, the second interpolates guide curves, and the third term interpolates the network of curves.

Further, the authors show how they achieve the fulfillment of all necessary conditions for a grid of curves using approximation, reparametrization. To do this, they use the Lagrange multiplier method, however, the approximation can rearrange the curves in such a way that the discrepancy for the final Gordon surface will be too large.

The article presents the results of this algorithm with examples on aircraft parts. This algorithm looks valid and working, but the probability of a significant error does not allow us to fully trust this method.

3.1. The algorithm for Coons-Gordon patches in Analysis

The algorithm for constructing bi-linear Coons patches (global approach) in Analysis Situs project based on such libraries as Open CASCADE library [11], Mobius [10] and TiGL [9]. It consists of the next several steps now:

- The user selects four edges. These edges are sorted so that a Coons patch can be constructed on them, that is, so that the pairs of edges are co-directional, that is, $c\_0 \uparrow\uparrow c\_1$ and $b_0 \uparrow\uparrow b_1$. The example of set of edges is shown in the figure 3.

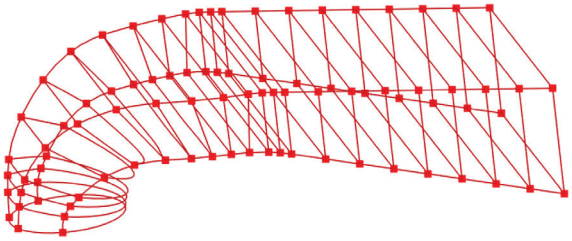

Figure 3: A network of curves

- Input curves are re-approximated. This is necessary so that the degrees of all curves are equal to three.
- Coons patches are constructed (see the figure 4).

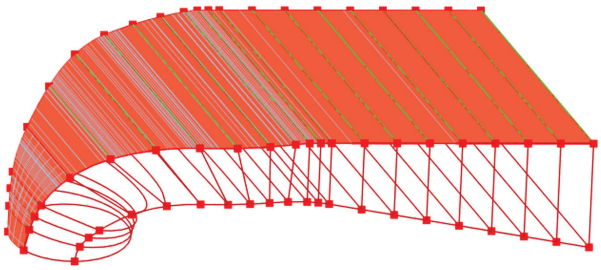

Figure 4: Coons patches from Analysis Situs

- The user specifies the number of intermediate isoparametric curves that are used to build the Gordon surface.
- Curves are projected onto the surface. The Gordon surface is built on the projected curves.

Thus, from the original four edges, we get the Gordon surface (fugure 5).

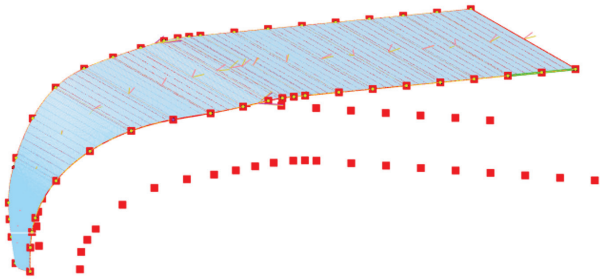

Figure 5: Surface from Analysis Situs

This algorithm uses re-approximation, which leads to large errors in the subsequent construction of surfaces. Another problem of this approach is the difficulty of working with models whose curvature of the resulting surfaces is steep; on such models, the algorithm either does not work or works poorly. Therefore, it is necessary to improve the original algorithm (figure 6).

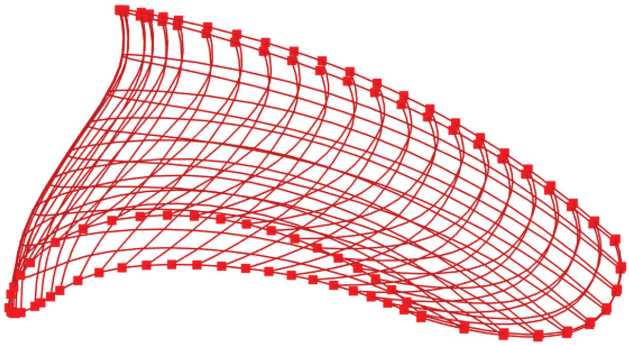

Figure 6: An example of the algorithm that does not work correctly

Algorithm for improving the construction of the Coons and Gordon surface:

- The user selects edges (curves) projected onto the desired surface.
- For each curve there are points of intersection with other curves and it is cut at these points.
- On the entire new set of cut curves, we find groups of four curves that form a closed curve (patch). For each set of four curves:
  - The curves are sorted so that a Coons patch can be constructed on them, that is, so that the pair of edges is co-directional, that is, $c\_0 \uparrow\uparrow c\_1$ and $b_0 \uparrow\uparrow b_1$.
  - A Coons patch is being constructed.
- Connect the resulting Coons patches and get one final surface (figure 7).

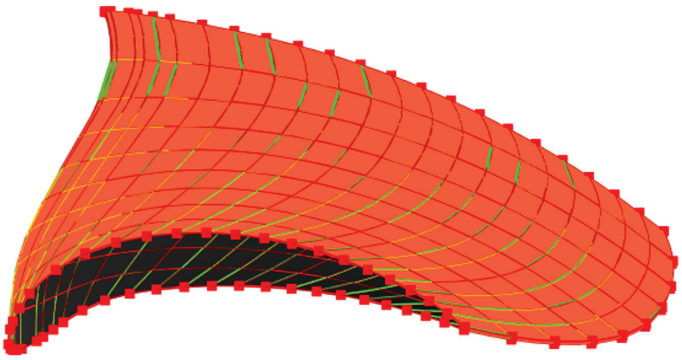

Figure 7: Coons patches

This algorithm does not include re-approximation and breaks the process of building one surface into building many small ones, which means the ability to work with curves of various degrees and a smaller discrepancy between the reference and final surfaces.

4. Conclusion

We have presented implementation of the local approach of Coons-Gordon surface algorithm. The Coons method, the Gordon Boolean sum and the global and local methods for constructing a surface from a network of curves were briefly described.

The global and local approaches are developed in open-source project Analysis Situs. The need of joining Coons patches appeared not only in Analysis Situs. For instance, in the work of Zhang, Yin and Yan [7], the authors analyze the process of constructing reservoir surface models using various algorithms, including Coons patches. The authors note that the positive feature of this algorithm is the need to have only four curves and the possibility of more flexible updating of models when new structural patterns are discovered during reservoir development. The result of this work is the generation of a 3D mesh from a set of surfaces. However, the authors noted that at the moment there is no open source project that would allow building such a grid from a given set of Coons patches. So our implementation can solve this problem as well.

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