Problems of Graphic Design of the Results of a Geometric Experiment

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Abstract

The article is devoted to the problems of graphical representation of drawings for purposes of interactive automated constructive geometry modeling. The authors demonstrate the necessity for studying principles of models creation to develop special informational technologies for visualizing, interpretation and processing of geometrical data. The article presents a retrospective review of methods for displaying geometric information, adopted among specialists in constructive geometric modeling. It is shown the importance to support scientific research in a field of geometry modelling, especially for multidimensional space for science, education and applied tasks implementation. The article formulates the goals and objectives of the further development of the system of constructive geometric modeling Simplex.

Keywords

Geometric modeling, interface design, geometric experiment, simplex.

1. Introduction

In the community of specialists involved in the study and design of constructive geometric models, over many years of research, certain ideas have developed about the styles and methods for presenting the results obtained both in symbolic and graphical form. The language of professional communication has been formed. An analysis of literary sources published in different years and in different regions of the world shows that, in general, these styles and methods have similar features and are based on a form of notation generally accepted in the international mathematical community that accompanies scientific information. However, unlike the analytical branch of mathematics, its synthetic branch – geometry – developed in a non-uniform way in the world. Its features lie, first of all, in the active support of the results obtained with illustrative material in the form of graphic diagrams and specific drawings based on the provisions of projective geometry [9]. There are no rigidly regulating standards in this area, and the ways of presenting material differ both in the regional plan and from the traditions inherent in individual scientific schools.

Active work on constructive geometric modeling [21–35] was carried out in the former USSR, mainly in the 50–70s of the last century, in particular, multidimensional [9, 11, 18, 20, 25]. It was during this period that many scientific papers on this topic were published, after which a decline in scientific activity began to be observed. Most of the references given in this article refer to this period. It may seem strange, but it was the progress in the field of computer technology that became the factor that began to suppress geometric thought, which did not find proper support in the emerging new information technologies.

The importance of introducing informatization tools in geometric modeling was noted by many authors. For example, in [32] the following is noted. “Scientific research and engineering developments aimed at developing the method of direct geometric modeling based on integrated systems of machine geometry and graphics are topical. The creation and application of such systems currently determines the main content of work on the automation of geometric modeling. These works are aimed both at improving the integrated systems of computer geometry and graphics themselves, and at developing the methodology of geometric modeling from the standpoint of modern computer science. An important

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aspect of this activity is the development of appropriate courses.” Unfortunately, the level of development of information technology did not allow for the full implementation of these ideas, specialists in the field of geometry were not professionally ready for informatization, as a result of which the development of the scientific discipline began to decline.

We have to admit that the crisis that broke out hit both geometric science and geometric education painfully. In our time, there is an understanding that the situation must be urgently corrected. Work on the subject of constructive geometric modeling continues [1, 2, 5, 6, 36–47]. Relentless attempts are being made to revive geometric knowledge in higher education. The works of colleagues who work in this difficult direction deserve great respect [8, 10, 12–18]. This article does not aim to evaluate these results, but if we at least briefly characterize them, we have to state that there is no clear understanding of the object, subject and method of constructive (including descriptive) geometry. And out of this misunderstanding come the results. The research conducted by the authors is based on the fact that the object of constructive geometry is understood as a form in all its manifestations, the object is the display of forms, and the method is projection schematism.

The article brought to the attention of readers concerns the development of an automation system for constructive geometric modeling in the context of creating interfaces for such systems. These issues, along with many others, are important, since their successful solution should ensure the continuity of technologies for working with geometric data and create conditions for reducing labor intensity and preventing the information collapse that inevitably accompanies traditional manual methods of geometric modeling. The development of new information technologies on this basis, which make it possible to preserve and increase the existing, but not fully demanded scientific experience, is an important task.

2. Problems of representation of geometric data

The requirements for the preparation of drawing and graphic documentation, regulated in national and international standards, are intended to support design activities and, therefore, cannot always be effectively used for the production of documentation accompanying geometric studies. Differences in the target orientation of the compared areas of activity cause difficulties when trying to use the means of graphic subsystems of design CAD systems to automate the work of a researcher dealing with geometry. For example, there are many examples of attempts to use the domestic design system Compass to simulate the graphical solution of descriptive geometry problems [18]. However, with a seeming increase in the efficiency of assimilation of the material by students through the introduction of modern information technologies, in essence, nothing is achieved in principle. The process remains very time-consuming, the prospects for its practical application in practice in the future are completely unclear. And most importantly, the algorithmic essence of the studied geometric algorithm is lost, which, in fact, is the subject of the discipline as a modeling tool that is universally applicable in solving many theoretical and practical problems. Design systems are practically not suitable for information support of the processes of solving more or less serious constructive geometric problems.

It should be noted that graphic design process automation systems can contribute to the process of documenting the results of geometric modeling, since they have a significantly greater freedom in choosing tools to achieve expressiveness. But in this case, the problem is only partially solved. Simplifying the preparation of materials for the production of presentations and the preparation of printing materials, they cannot provide a solution to the problems of fully revealing the algorithmic essence of constructive geometry. Moreover, freedom should be used reasonably, the free and unsystematic interpretation of geometric entities through images can lead to the complexity of the perception of scientific ideas by members of the geometric community and other consumers of geometric information. We add that graphic design systems cannot provide the applied application of the geometric models they represent.

All of the above speaks of the development of information systems and technologies with a certain specialization, which together solve the problems of both the synthesis of models and their presentation for various consumer purposes. Such systems exist and are collectively known as computer geometry systems. The most famous representative of such systems is the Geometer's Sketchpad software package from KCP Technologies, which allows specialists in the field of constructive geometry to solve their main tasks: to model, experiment, share research results with colleagues, prepare results for publication, etc. In particular, the well-known WEB-projects Encyclopedia of Triangle Centers Encyclopedia of Quadri-Figures, Encyclopedia of Polygon Geometry.

The domestic analogue of this development is the Simplex system [5], the concept of which and the software implementation are developed by the authors of this article. Performing somewhat similar functions, the Simplex system is based on other ideas about constructive geometry. Its ideology is largely based on the theoretical principles formulated by the Russian geometric school, in particular, on the ideas of multidimensional geometric modeling. An analysis of literary sources shows that despite the understanding of the need to build models for the interpretation of multidimensional data in Western countries, the geometric theory of this scientific direction is not developed there. As a result, systems like Geometer's Sketchpad cannot fully support all aspects of designing, documenting, and implementing constructive geometric models, especially models based on planar-projection principles.

In the foregoing, the system should provide the solution of the following tasks:

1. Model development and debugging
2. Solving problems on the model
3. Presentation of the model as a project
4. Implementation of the geometric experiment
5. Representation of the model in the form of a graphic document
6. Automated synthesis of mathematical notation
7. Synthesis of the executable program
8. Model export functions for various means of visualization and information transfer
9. Network dissemination of information and knowledge
10. Contribute to the improvement of the theory of constructive geometric modeling
11. Revival of the accumulated experience of geometric science, support for publication activity, patenting of algorithms and programs.

2.1. Object Display

It is not possible to make a comprehensive overview of the ways in which geometric data is represented in drawings. Some examples given in this article are based on the personal experience of the authors and on the experience of developing the Simplex constructive geometric modeling system, but they, of course, cover only some trends and do not claim to be an exhaustive analysis of the problem. However, the study of this aspect of the activity of geometry researchers and technology developers seems to be extremely important, since taking into account the factors of naturalness of work in the geometric design environment contributes to the development of the interface of geometric modeling automation systems, and, as a result, the improvement of instrumental support and the development of scientific discipline. This ensures the simplification of work on documenting research results, textual and graphic support for scientific and educational publications, and dissemination of geometric knowledge.

Design models are inherently based on symbolized images, i.e. each model object is given an individual name that distinguishes it from other objects of the same model. The naming must be the same both in the drawings and in the accompanying text explaining the content of the model. Constructive modeling, which associates a drawing with an active cybernetic device that processes information, considers each object of the model as a variable, which makes it possible to compare the process of naming objects with the naming of variables in programming languages and apply a similar notation for this. The conclusions that served as the basis for the implementation of the principles of naming and displaying objects of the system of constructive geometric modeling are made on the basis of an analysis of many sources of both domestic and foreign origin. In geometry, a single object is usually denoted by a letter (rarely by several letters). Object types are usually distinguished both by lowercase or uppercase characters, and by belonging to different alphabets (usually Latin and Greek). The names of names, depending on their functional or semantic purpose, may include indices. Typically, a subscript is used for the following purposes: indexing objects that are related in meaning, indicating that an object belongs to the field of projections with a common part of the generic name, to indicate the object of influence on the designated object, etc. The superscript is usually associated with the dimension of the space, the number of the image of the object, if it is the result of repeated execution of operations of the same type on some preimage, and so on. Indexes can contain both numeric and alphabetic indexes, consist of groups of objects separated by commas. Sometimes special characters are used in indexes, like plus, minus, asterisks, and some others. Indexing of indexes is extremely seldom applied. Indexes leading before the name are not found in the literature. In the drawings, the naming of objects as the results of the operation does not occur, however, such designations are appropriate for shortening textual explanations for the drawings. In a number of cases, for naming images with a common generic name of the prototype, a notation with one or more apostrophe strokes (up to four), as well as Roman numerals [26] is used. For the same purposes or for the purposes of indicating the conjugacy (complementarity and other purposes) of objects, superscript strokes (no more than two) are used [23]. Sometimes apostrophe strokes denote the names of object projections.

2.2. Geometric drawing as an applicative display model

A constructive geometric model implements a relational scheme that reflects the process of generating some objects from others. The design of the model, in whatever sequence it is carried out, establishes causal relationships between the objects of the model, as a result of which the model can be considered as an abstract device operating in time. It is clear that different types of geometric objects are formed at different times, however, the "temporal" way of displaying them in a document will not comply with the generally accepted norms for representing geometric information, since the images would overlap each other. The perception of the drawing in this form is inconvenient, therefore, in the practice of creating graphic documents, the so-called. application way of presenting data, and different types of data have individual display priorities. So, for example, all images that have internal filling are displayed first, linear images - after them, and only after that point objects are displayed. As practice shows, a similar principle should be strictly observed in the development of geometric modeling automation systems, both when creating an interface and when developing documentation systems.

2.3. Points

The geometric object "point" is usually denoted by an uppercase Latin letter. This principle is in good agreement with the rules for naming variables in programming languages such as C. When there are a large number of points in a computer-implemented model, it makes sense to use more characters in the name, leaving the first character in uppercase, although this method is not found in the traditional practice of documenting geometric drawings. Sometimes points that have an auxiliary value are denoted by a number. When implemented on a computer, this method conflicts with the principle of notation of numerical constants, however, if you supply a number, for example, with a leading underscore that will not be displayed in documents, this problem is eliminated.

Small circles are usually used as graphic markers depicting points in drawings. When documenting on information carriers, their purpose is to interpret the positional relationships of points and their semantic purpose. In computer geometry systems, point symbols perform both display functions and means of interface interaction with a computer model (functions of selection, identification, changing values, etc.). For the semantic distinction of points, circles are used without filling the inner area, with a filling identical in color to the document carrier, a filling identical in color to the outline of the circle. Also used are markers such as a circle with a dot inside [24], a circle with concentric inner circles (up to two), circles with small outer strokes [34]. Multiple markers can be used in drawings. Modern computer technology allows the use of markers of different colors, but if it is necessary to publish a document in black and white, the value of this method becomes doubtful. In modern infographic images, point markers of various shapes are widely used, but such markers are not used in the literature on constructive geometry. Apparently, it makes sense to equip computer geometry systems with the ability to use some sets of markers that differ in shape from a circle, but their number should not be large, and they should be well distinguishable from each other at relatively small sizes. The labels of point names are displayed near the markers strictly horizontally. If it is impossible to place the inscription next to the point, leader lines are used.

Imaginary points, the use of which in the conditions of computer simulation acquires an important meaning, cannot be depicted in traditional drawings and are interpreted only indirectly through the objects that generate them. With a computer implementation, such points can be conditionally represented in the form of special marker images and play a full-fledged role as an interface for interacting with the model.

2.4. Straight lines

The geometric object "straight line" is usually denoted by a lowercase Latin letter. Much less often, by means of letters corresponding to the names of the points through which this line passes. Otherwise, the principle of naming straight lines (as well as other lines in general) is similar to the methods of naming points.

As graphic markers depicting straight lines in the drawings, lines of various thicknesses and styles are usually used (dashed, dash-dot to indicate axes), with arrows at the end (to indicate direction), with arrows in the inner area (to indicate the sequence of actions), trapezoidal lines [34], lines with perpendicular strokes [30, 33], crossed straight lines [35], and a number of others. To ensure the possibility of some linear images from others or, on the contrary, to mark their similarity, special marks are used [22].

It is proposed to represent the images of imaginary lines in the form of a conditional image - marks [37].

Being an unrestricted geometric object, lines can make a drawing difficult to visually perceive and break its aesthetics. To eliminate this shortcoming, when displaying them, it is necessary to take into account the location of the points incident with these lines and to draw only within the limits of the incident objects corresponding to them. A feature of the representation of straight lines in the drawings of projection models is the possibility of obtaining images that go beyond the provided area of the drawing, within which most of the geometric images of the model have an acceptable display scale. An example of a way to implement a drawing, for example, may be the technique proposed in [21].

Link lines are auxiliary lines that indicate the projection connection of the projections of points located in separate fields on the drawings of planar projection models. Communication lines are a specific visualization feature of such models. They do not belong to the fields, but characterize the connection of the fields and indicate the direction to the projection centers. In the drawings, these lines are marked with a single symbol for all of them, in the superscript of which the name of the point being modeled is indicated. Communication lines are drawn with a solid thin line. With a large number of communication lines, they can unnecessarily fill the space of the drawing and make it unsuitable for perception. In such cases, it is advisable to depict the communication lines as strokes that intersect the projections of points only in a short interval. Communication lines can be supplied with arrows explaining the progress of solving the problem.

A specific feature of flat projection models is also the axial lines of the projection fields. Such lines are usually called axes by a letter with field numbers, since they belong to both fields at the same time. The axes are usually drawn as a dash-dotted line, either crossing all connection lines, or as a separate solid stroke with the obligatory indication of the axis name. In cases of standard arrangement of views, centerlines are not drawn.

2.5. Circles and conics

A feature of solving problems using projection methods is the formation of circles of zero and imaginary circles and conics and the need to work with such images. It was not possible to find any graphical ways of representing such objects in the geometric literature. There are only indirect references to them through accompanying real images.

2.6. Transformations, integrated images

Geometric transformations, such as projectives, collineations, inversions, etc., are the most important tool for solving geometric problems with projective content. Transformations do not have any visual-graphical interpretations, perhaps, except for their indication through fiducial elements. However, this way of interpreting transformations cannot be considered convenient. Therefore, these images in the literature are denoted either by the corresponding text symbols on the drawing field, or, in extreme cases, with the help of conventional graphic symbols [28]. Also noteworthy are ways of displaying collections of elements, such as rows and bundles [30, 33].

2.7. Formation of a text accompanying document

Accompanying the developed drawing with explanatory text is a very time-consuming process. Its complexity lies in the need to strictly follow the sequence of actions, the rules for naming and compiling records of geometric operations, the reality or imaginary values, complementarity, the designation of models and spaces, etc. [24, 25, 27, 28, 29, 31]. The problem is exacerbated by the specifics of designating objects in projection fields, the need to take into account their visibility and other attributes. Naturally, such a formal but necessary stage requires automation.

The desire of the researcher to provide the results of his activity with practical significance and turn it into a calculation and graphical tool prompts him, in the end, to translate it into an analytical form for the purpose of programming and implementing it with computer mathematics support tools. If in the recent past this was achieved as a result of programming in a general technical language, now Mathcad (PTC) and similar systems are used for these purposes [7]. The problem here is that converting a drawing into an analytical form is done manually and involves many errors, despite the fact that this action is formal. Accordingly, these tasks require the development of a special interface.

2.8. Features of the interface serving the process of designing discrete-continuous models

The principle of constructing geometric models representing multidimensional data is to reduce the dimension of the original operational space by performing the operation of its projection onto spaces of lower dimension (picture spaces). Projection centers and picture spaces in the aggregate are called the apparatus for projecting a model of a multidimensional space: it is meaningless to talk about building a model without choosing a projection apparatus. Based on the requirement to comply with the isomorphism of the resulting model in the whole operational space, projection is carried out by choosing the necessary and sufficient number of projection centers and the corresponding number of picture spaces onto which projection is carried out from these centers. It should be specially noted that the isomorphism of modeling is achieved precisely in general, because during modeling, excluded spaces will always be found, the representation of which by means of the model is impossible. In turn, any model will contain the so-called excluded images of the model, which do not unambiguously represent anything in the simulated space, but perform an important organizational function in the model. The variety of models is determined by the variability in the choice of the number and types of projection centers and, accordingly, picture spaces.

The picture spaces included in the model are in a certain sense not autonomous, since the modeling of the images of the operational space in them is subject to the laws dictated by the choice of the projection apparatus. The subordination of picture spaces to a single model does not allow them to be treated as independent spaces, and therefore, terminologically, such spaces are usually called model fields. As a result of building a multidimensional space model, images of objects contained in the operational space are formed in the fields. Thus, the so-called procedure is performed. external modeling. In turn, the nature of the relationship of images formed in the model and arising as a result of the subordination of fields can be expressed by an independent procedure that does not require reference to the original projection apparatus. Formal actions with images performed exclusively within the framework of the model are the subject of the so-called. internal simulation. Revealing the essence and regularities of the apparatus of internal modeling in the fields of the model is the subject of scientific novelty in the construction of each new previously unknown geometric model.

The subordination of fields, however, does not limit the freedom of their transformation. In particular, in the practice of geometric modeling, the combination of several fields within a single space is widely used (for example, through collinear transformations that combine field spaces with each other). Such a combination provides certain instrumental conveniences when working with the model, since due to it. It is possible to reduce the required number of information carriers on which the model is materially implemented and documented, and it is easier to trace the connections that make up the subject of internal modeling. However, the same action requires the availability of means to distinguish between objects located on the same carrier, but belonging to different fields. In the Simplex system, such means are the attributes of matching an object with fields and tools for displaying fields.

It is impossible to formulate any fundamental restrictions that affect the choice of dimension of picture spaces. It is only important that this dimension be lower than the dimension of the modeled operational space. However, in practice, projection models are most widely used, in which picture spaces have the second and first dimensions. This is primarily due to the fact that the graphic documentation of the model on a material information carrier is associated with a flat sheet of paper or a monitor screen. Reducing the dimension of the picture space to one is the subject of analytical modeling and is associated with the concept of number. The current appearance of virtual and augmented reality devices allows us to hope that with their help it will be possible to practically realize the picture spaces of the third dimension, as a result of which the development and construction of the corresponding models may be of known scientific interest.

The latest version of the Simplex system allows to define up to nine fields in models. Field attributes are designated by Arabic numerals from 1 up to 9. Each object of the system has a sign (attribute) belonging to the field. Consequently

- an object may not be assigned to any field at all;
- an object can only be assigned to one field;
- an object can be assigned simultaneously to several fields at once.

The geometric construction display window has visual buttons, activation of at least one of which switches it to the geometric construction display mode, taking into account the attribute of a particular field.

In the event that none of the buttons for switching the display window of the geometric construction to the display mode taking into account the characteristics of the fields is not activated, all objects, regardless of the way they are related to the fields of the model, are displayed in their usual form. All of them can be selected with the Pick and Shaper tools.

In the event that one or more buttons for switching the display window of a geometric construction to the display mode taking into account fields' prizes are activated, all objects, attributes of correlation with fields that correspond to the numbers of activated fields, as well as objects that are not correlated with any field, will be displayed in usual form. All others will be displayed in Watermark mode. Only those objects that are displayed in the normal view can be selected with the Pointer and Shaper tools, all other objects do not interact with these tools.

Based on practical experience in solving problems related to the synthesis of discrete-continuous models and designing a user interaction interface with the Simplex system, the authors managed to formulate rules for correlating objects with fields and tools for assigning corresponding features that greatly simplify this process.

The rules for automatically assigning attributes for matching objects with fields are heuristic in nature: they were formulated on the basis of a generalization of experience with geometric models, in which the distribution of objects by fields is dictated by the practical expediency of such a presentation of information in graphical form.

The automatic assignment of the attribute is performed either when entering an operator using the accelerator keys, or at the moment the command is completed using the operator input panel. This or that attribute value is determined by the following list of rules:

- If the command is entered at the moment when no field selection button is activated in the geometric construction display window, the objects presented in the output parameters of the operator being created do not receive a sign of belonging to the field, even if among the objects presented in the input parameters there are those that have a sign of matching with some fields.

- In the event that any field selection buttons are activated in the geometric construction display window, then the objects presented in the output parameters of the created operator will be compared with the numbers of the selected fields, provided that among all the objects presented in the input parameters there are no signs of matching with any fields.
- In the event that any field selection buttons are activated in the geometric construction display window, then the objects presented in the output parameters of the operator being created will be compared with field numbers that are common to all objects listed in the input parameters (even if the button of some or the general field is not activated), except for those that are not assigned to any field. If there are no common matching numbers, then the objects presented in the output parameters of the generated statement are not matched with any field numbers.

3. Conclusion

The following scientific results were obtained in the research:

1. The analysis of literary sources on the subject of constructive geometric modeling was carried out and the systematization of the graphic elements present in the drawings of geometric models was carried out.
2. A set of recommendations has been developed for the development of an interface for a geometric modeling system that implements a multi-picture data representation
3. It is demonstrated the need to develop a specialized system that allows you to implement a full range of tasks for designing and documenting constructive geometric models.

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