Reversibility of Modified Cyclographic Projection

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Abstract

In cyclographic modeling of lines of space R^3 direct and inverse problems and their solutions are known. In addition to the classic cyclographic projection, there is modified cyclographic projection of a line of space, developed on the basis of the classic one. The modified cyclographic projection has practical relevance in design of general purpose road surface forms. For this projection, the solution of the direct problem, i.e. to determine modified cyclographic projection given a curve of space (road axis), is known. In this paper, we propose a solution to the inverse problem - to restore the initial space curve given its modified cyclographic projection. The paper provides justification for solution to the inverse problem and considers on example an analytic solution to the inverse problem given second-order curves. The results of this study can be applied as the basis for development of computer-aided design systems for road surface forms, both in creation of new working surface forms, and in restoration the existing ones.

Keywords

Geometric modeling, cyclography, modified cyclographic projection, inverse task, road surface form design.

1. Introduction

Due to advances in information technology, computer aided design and computer algebra systems, the method of cyclographic modeling of geometric objects is more and more often finding application in solutions to completely different areas of both science and production [1-4]. One of the distinguishing features of this method is the parametric bijective correspondence of the triad of curves generated by cyclographic mapping of a spatial curve into projection plane. This fact allows one to solve both the direct and the inverse problems of cyclographic modeling of a spatial curve.

The development of cyclographic model of a spatial curve allowed the authors to acquire its modified cyclographic projection, where the straight generatrices of the ruled surfaces constituting the cyclographic modeling apparatus elements are located in a plane perpendicular to the orthogonal projection of the curve [4]. In further studies, such projection was applied in development of the geometric model for common purpose road surface formation [4].

Analysis of the body of research in the area of automated design of road surface forms allows us to conclude that over the past years a relatively new method of design has emerged. This method is based on 3D representation of the modeled objects of road construction. Retirement of the traditional biplane design allows one to significantly simplify and reduce computational operations in design, as well as visualize the modeled road surface objects on each stage of design. At the same time, we see the appearance of new problems that require finding the location of road axis in space given various blueprints, plans or photographs. Even though in road surface form design based on the classic cyclographic mapping this problem is effectively solved [1,5], in the case of the modified cyclographic mapping behind the geometric model for common purpose road surface formation this problem has not yet been studied.

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2. Theory

In cyclographic mapping a projection of a point $A(x, y, z)$ of space R^3 constitutes a cycle (a directed circle) in plane $z = 0$ of radius z centered at coordinates (x, y). The direction of the cycle is determined by the location of the point with respect to projection plane, i.e. the sign at z coordinate [5-8]. This method of mapping generates a projecting cone with vertex at the point A and base in plane $z = 0$. Its base is the cycle, and the cyclographic projection of the point. Evidently, the cyclographic projection of a spatial curve $\overline{P}(t) = (x(t), y(t), z(t))$; $\overline{P}'(t) \neq 0; t \in R : T_0 \leq t \leq T$ constitutes an envelope of a one-parameter multitude of cycles consisting, in general, of two effective branches. The given spatial curve and its cyclographic projection combined allow us to acquire a ruled surface, where these curves serve as generatrices. In classic cyclographic projection half-angle α at the projecting cone vertex is equal to $\alpha = 45°$; the equations for the cyclographic α-projection of a spatial curve are known [7]. In their previous studies, the authors have derived the equations for the cyclographic β-projection and $\beta(t)$-projection of a spatial curve. Here half-angle β can either have constant value in range $0° < \beta < 90°$, or even be represented by a certain function $\beta(t)$ of the initial curve parameter [4]. In the latter case, the generalized cyclographic projection is described by the equations of the following form:

$$x_{\beta(1,2)}(t) = x(t) + z(t) \cdot e(t) \frac{-x'(t) \cdot \mu(t) \mp y'(t)\sqrt{\lambda(t) - \mu^2(t)}}{\lambda(t)},$$
$$y_{\beta(1,2)}(t) = y(t) + z(t) \cdot e(t) \frac{-x'(t) \cdot \mu(t) \pm x'(t)\sqrt{\lambda(t) - \mu^2(t)}}{\lambda(t)}, \qquad (1)$$

where $\mu(t) = e(t) \cdot z'(t) + e'(t) \cdot z(t)$; $\lambda(t) = x'(t)^2 + y'(t)^2; e(t) = tg(\beta(t))$.

The acquired generalized equations for cyclographic projection of a spatial curve have laid the basis for development of the geometric model of road surface formation through cyclographic mapping [4]. However, state codes and standards dictate certain requirements to designed objects of road construction [9]. This includes the requirement for all orthogonal projections of straight generatrices of road surface form to be located in a plane perpendicular to the orthogonal projection $\overline{P}_1(t)$ of road axis $\overline{P}(t)$. In order to conform to this requirement, the authors have proposed modification of the classic cyclographic projection that rotates (see Figure 1) the orthogonal projections into the desired location. This transformation is considered in detail in paper [4]. The acquired equations for the modified cyclographic projection of a spatial curve are of the following form:

$$x_{ch}(t) = x(t) \mp \frac{y(t)' \cdot (z(t) \cdot e(t))}{\sqrt{(x(t)')^2 + (y(t)')^2}},$$
$$y_{ch}(t) = y(t) \pm \frac{x(t)' \cdot (z(t) \cdot e(t))}{\sqrt{(x(t)')^2 + (y(t)')^2}}. \qquad (2)$$

As follows from the theory of bisectors [10], the orthogonal projection $\overline{P}_1(t)$ of curve $\overline{P}(t)$ constitutes a bisector line for the curves $\overline{P}_{\beta(1)}(t)$ and $\overline{P}_{\beta(2)}(t)$. It is known that a bisector line of two geometric objects constitutes a geometric locus of points equidistant to the points of the objects, where the distance to the bisector line is determined in direction orthogonal with respect to the objects [10, 11]. Obviously, the curve $\overline{P}_1(t)$ does not constitute a bisector line of the modified cyclographic projection curves $\overline{P}_{ch(1)}(t)$ and $\overline{P}_{ch(2)}(t)$. According to the construction scheme, the curve $\overline{P}_1(t)$ constitutes a line of curvilinear symmetry, since normal distances from any point of curve $\overline{P}_1(t)$ to curves $\overline{P}_{ch(1)}(t)$ and $\overline{P}_{ch(2)}(t)$ are equal.

Theoretically, is it at all possible to reconstruct the initial curve given its modified cyclographic projection? If we consider the problem $\overline{P}(t) \to \overline{P}_{ch}(t)$ as direct, then obviously, $\overline{P}_{ch}(t) \to \overline{P}(t)$ would be the inverse problem. The present paper is dedicated to solution of this problem. It is worth noting that, apart from the theoretical interest, the inverse problem can find application in reconstruction of road axis given various design documents, blueprints or real-life measurements of road edge reference coordinate points [12-14]. The latter is most relevant in road surface restoration, when, as a rule, the actual position of the road axis is compared to its design position in order to estimate the volume of repair operations [12]. Of high importance is the 3D visualization of the design solution allowing one to estimate spatial visibility on the road, visibility on turns and joints, etc.

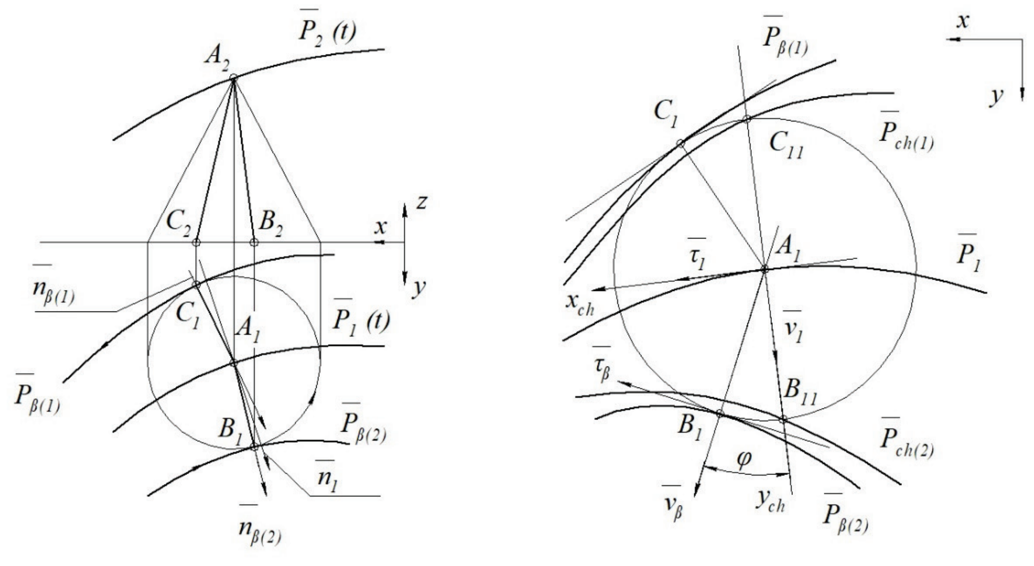

Figure 1: Modified cyclographic projection $\overline{P}_{ch(1,2)}(t)$ construction scheme [4]

Let us consider solution of the inverse problem $\overline{P}_{ch(1,2)}(t) \to \overline{P}(t)$, where $\overline{P}_{ch(1,2)}(t) : \overline{P}_{ch(1)} \cup \overline{P}_{ch(2)}$. Note that parameters at the given curves $\overline{P}_{ch(1)}$ and $\overline{P}_{ch(2)}$ aren't the same: $\overline{P}_{ch(1)} = \overline{P}_{ch(1)}(t_1), T_{10} \le t_1 \le T_{11}$, $\overline{P}_{ch(2)} = \overline{P}_{ch(2)}(t_2), T_{20} \le t_2 \le T_{22}$. It is therefore required to determine the relation of the parameters of the given curves $\overline{P}_{ch(1)}(t_1)$ and $\overline{P}_{ch(2)}(t_2)$. In classic cyclographic modeling this task is known as the inverse problem; its solution is known [2, 9]. Let us briefly consider the idea behind this solution. Given two branches $\overline{P}_{\beta(1)}(t_1)$ and $\overline{P}_{\beta(2)}(t_2)$ of cyclographic projection in projection plane $z = 0$, it is required to reconstruct the corresponding curve $\overline{P}(t)$ in space. In order to do that, curves $\overline{P}_{\beta(1)}(t_1)$ and $\overline{P}_{\beta(2)}(t_2)$ are put into correspondence with respective cyclographic images constituting ruled surfaces $\Phi_{\beta(1)}$ and $\Phi_{\beta(2)}$ with a certain incline of generatrices to plane $z = 0$. For example, in *α*-mapping the incline angle is equal to 45°, while in *β*-mapping the incline angle is (90 - *β*)° [2]. Then the curve of intersection of these surfaces is found analytically, expressed, for instance, through parameter t_1. In practice, analytic solution to this problem can be acquired in case both $\overline{P}_{\beta(1)}(t_1)$ and $\overline{P}_{\beta(2)}(t_2)$ constitute second-degree curves or outlines of such curves. Therefore the curve $\overline{P}(t_1)$ (Figure 2) as the orthogonal projection of the curve $\overline{P}(t) : \Phi_{\beta(1)} \cap \Phi_{\beta(2)}$ constitutes the sought curve. This allows us to bring the curve $\overline{P}_{\beta(2)}(t_2)$ to parameter t_1. As a result, cyclographic mapping in projection plane forms a triad of related curves $\overline{P}_1(t_1)$, $\overline{P}_{\beta(1)}(t_1)$, and $\overline{P}_{\beta(2)}(t_2 = f(t_1)), f^{/}(t_1) \ne 0$.

Obviously, the described solution does not work for curves $\overline{P}_{ch(1)}(t_1)$ and $\overline{P}_{ch(2)}(t_2)$ acquired through the modified cyclographic mapping. It is also obviously impossible to uniquely define the position of the sought curve $\overline{P}(t)$ without knowing the position of the curve $\overline{P}_1(t)$ of the cycle centers and the value of generatrix turn angle φ (see Figure 1b).

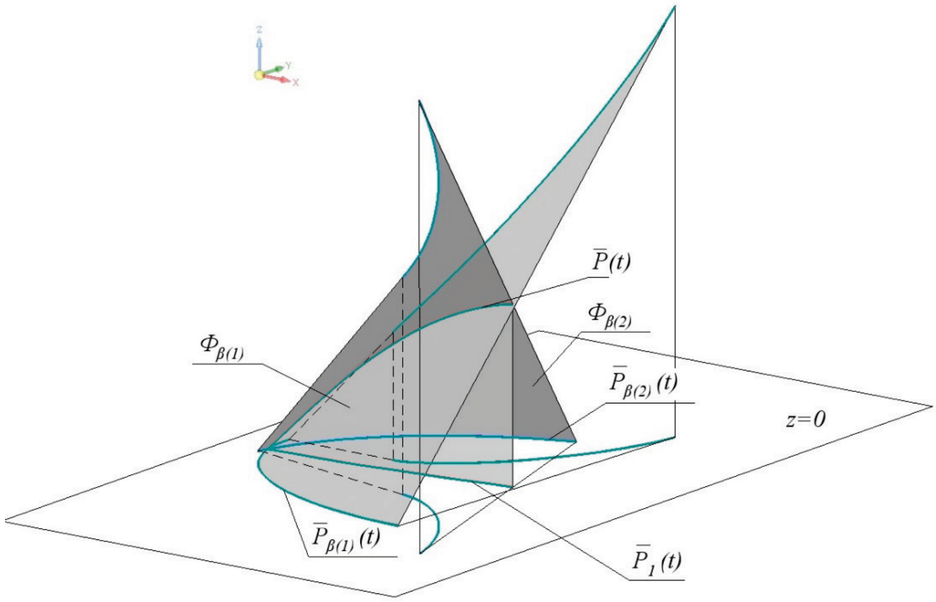

Figure 2: Visualized solution to the inverse problem of cyclographic modeling of a spatial curve

Construction of α-surfaces with generatrices $\overline{P}_{ch(1)}(t_1)$ and $\overline{P}_{ch(2)}(t_2)$ allows one to acquire the spatial curve $\overline{P}_{ch}(t_1):\Phi_{ch(1)}\cap\Phi_{ch(2)}$ and its orthogonal projection $\overline{P}_{ch,1}(t_1)$) (Figure 3). Obviously, the curve $\overline{P}_{ch,1}(t_1)$ is the bisector line of the curves $\overline{P}_{ch(1)}(t_1)$ and $\overline{P}_{ch(2)}(t_2)$, but it is not the line of curvilinear symmetry, i.e. not the orthogonal projection of the sought curve $\overline{P}(t)$. However, the curve $\overline{P}_{ch,1}(t_1)$ allows us to bring the given the curves $\overline{P}_{ch(1)}(t_1)$ and $\overline{P}_{ch(2)}(t_2)$ to bijective correspondence of parameters, i.e. to common parameterization. Now that any point C_{1i} of the curve $\overline{P}_{ch(1)}(t_1)$ is bijectively correspondent to a certain point B_{1i} of the curve $\overline{P}_{ch(2)}(t_2)$, it is possible to find the midpoints of straight line segments $B_{1i}C_{1i}$ constituting the points A_1 of the sought curve $\overline{P}_1(t)$ (see Figure 3). Since projecting cone base radius is equal to distance A_1B_{1i} (or A_1C_{1i}), it is also possible to reconstruct the spatial curve $\overline{P}(t)$ for the case of cyclographic α-projection.

3. Results of experiments

Let us consider an example. The two given curves $\overline{P}_{ch(1)}(t_1)$ and $\overline{P}_{ch(2)}(t_2)$ (Figure 4) located in projection plane $z=0$ are defined by the following equations:

$$x_{ch(1)}(t_1)=2-t_1^2/4; \qquad x_{ch(2)}(t_2)=-2t_2^{\,2};$$
$$y_{ch(1)}(t_1)=8t_1; \quad 0\le t_1\le 1. \qquad y_{ch(2)}(t_2)=3t_2; \quad 0\le t_2\le 1{,}5.$$

It is required to reconstruct the initial spatial curve $\overline{P}(t)$, for which these curves constitute the modified cyclographic projection.

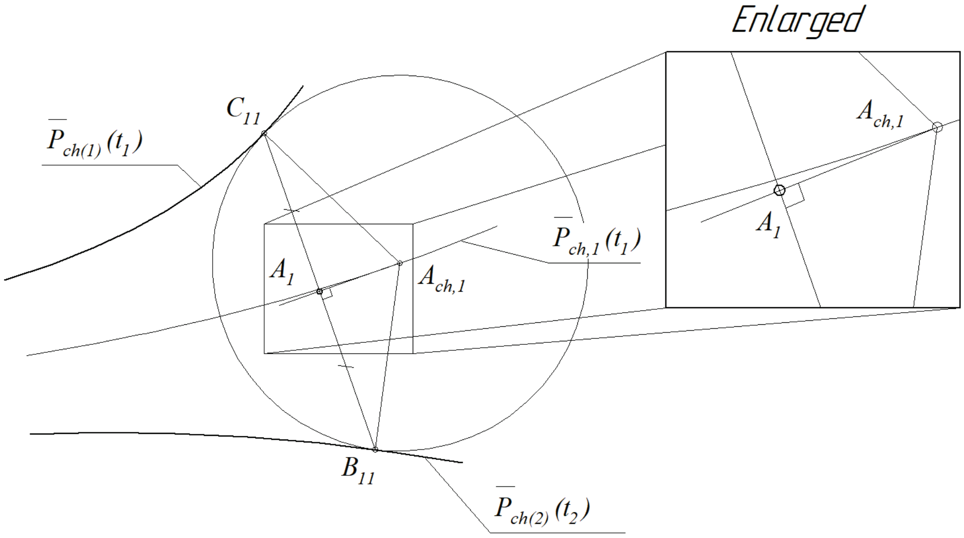

Figure 3: Scheme for reconstruction of points of the orthogonal projection $\overline{P}_1(t)$ of the initial curve $\overline{P}(t)$ given the branches of the modified cyclographic projection of $\overline{P}(t)$

Following the algorithm for the inverse problem of cyclographic modeling of a spatial curve, let us put the given curves into correspondence with respective α-surfaces [2,9]. To accomplish this, let us construct spatial images of the evolutes of these curves through the formulas known in differential geometry:

$$x_{E(i)}(t_i) = x_{ch(i)}(t_i) + y'_{ch(i)}(t_i)\frac{(x'_{ch(i)}(t_i))^2 + (y'_{ch(i)}(t_i))^2}{x''_{ch(i)}(t_i)\cdot y'_{ch(i)}(t_i) - x'_{ch(i)}(t_i)\cdot y''_{ch(i)}(t_i)};$$

$$y_{E(i)}(t_i) = y_{ch(i)}(t_i) + x'_{ch(i)}(t_i)\frac{(x'_{ch(i)}(t_i))^2 + (y'_{ch(i)}(t_i))^2}{x''_{ch(i)}(t_i)\cdot y'_{ch(i)}(t_i) - x'_{ch(i)}(t_i)\cdot y''_{ch(i)}(t_i)};$$

$$z_{E(i)}(t_i) = \pm\sqrt{(x_{ch(i)}(t_i) - x_{E(i)}(t_i))^2 + (y_{ch(i)}(t_i) - y_{E(i)}(t_i))^2},$$

where i=1,2. We do not cite the evaluation of the acquired equations due to its awkwardness.

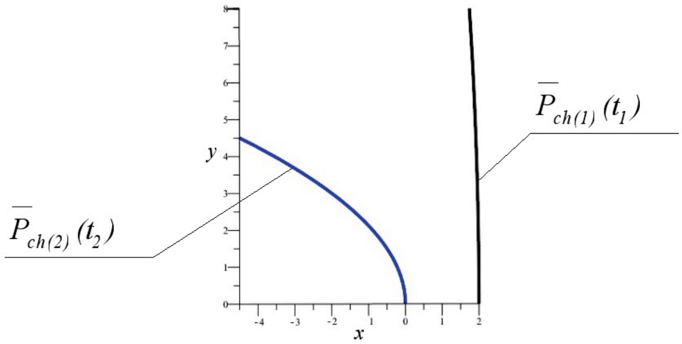

Figure 4: The given data for the considered example

The given curves in combination with the respective spatial images of their evolutes form ruled surfaces $\Phi_{ch(1)}(t_1)$ and $\Phi_{ch(2)}(t_2)$. The equations for these surfaces are of the following form:

$$X_{ch(1)}(t_1,l_1)=x_{E(1)}(t_1)+l_1\left[x_{ch(1)}(t_1)-x_{E(1)}(t_1)\right];$$
$$Y_{ch(1)}(t_1,l_1)=y_{E(1)}(t_1)+l_1\left[y_{ch(1)}(t_1)-y_{E(1)}(t_1)\right];$$
$$Z_{ch(1)}(t_1,l_1)=z_{E(1)}(t_1)(1-l_1);\quad 0\le t_1\le 1,\quad 0\le l_1\le 1.$$
$$X_{ch(2)}(t_2,l_2)=x_{E(2)}(t_2)+l_2\left[x_{ch(2)}(t_2)-x_{E(2)}(t_2)\right];$$
$$Y_{ch(2)}(t_2,l_2)=y_{E(2)}(t_2)+l_2\left[y_{ch(2)}(t_2)-y_{E(2)}(t_2)\right];$$
$$Z_{ch(2)}(t_2,l_2)=z_{E(2)}(t_2)(1-l_2);\quad 0\le t_2\le 1{,}5;\quad 0\le l_2\le 1.$$

By equating $X_{ch(1)}(t_1,l_1)=X_{ch(2)}(t_2,l_2)$, $Y_{ch(1)}(t_1,l_1)=Y_{ch(2)}(t_2,l_2)$, $Z_{ch(1)}(t_1,l_1)=Z_{ch(2)}(t_2,l_2)$, we can acquire a system of three equations in four unknown. Such system is solved through functional dependencies of its parameters. In the considered example, we start by expressing the parameter $l_2=f_2(t_1,t_2,l_1)$ from the equation $Z_{ch(1)}(t_1,l_1)=Z_{ch(2)}(t_2,l_2)$. Then, by substitution of the acquired expression into the equation $Y_{ch(1)}(t_1,l_1)=Y_{ch(2)}(t_2,l_2)$, we express the value $l_1=f_1(t_1,t_2)$. Further substitution of the acquired expressions for l_1 and l_2 into the equation $X_{ch(1)}(t_1,l_1)=X_{ch(2)}(t_2,l_2)$ allows us to express parameter $t_2=f(t_1)$. Substitution of the acquired dependencies $l_2=f_2(t_1,t_2,l_1)$, $l_1=f_1(t_1,t_2)$, and $t_2=f(t_1)$ into the equations for surface $\Phi_{ch(1)}(t_1)$ results in the parametric equations for the spatial curve $\overline{P}_{ch}(t_1)$:

$$x_{ch}=f_x(t_1);\ y_{ch}=f_y(t_1);\ z_{ch}=f_z(t_1),\quad 0\le t_1\le 1.$$

Reconstruction of the curve $\overline{P}_{ch}(t_1)$ is visualized on Figure 5. All the calculations of functional dependencies of parameters performed below, as well as all the visualizations presented on the figures, were performed in *Maple* computer algebra system.

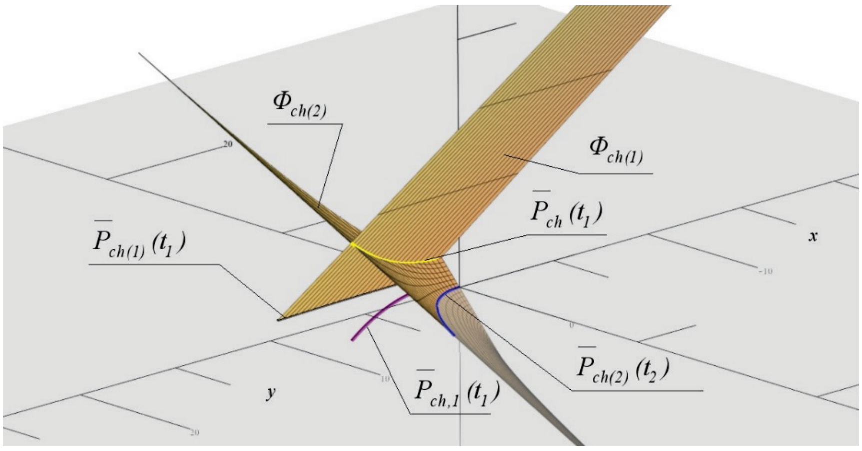

Figure 5: Reconstruction of the curve $\overline{P}_{ch}(t_1)$. Spatial visualization

The acquired curve $\overline{P}_{ch}(t_1)$ allows us to express the curve $\overline{P}_{ch(2)}(t_2)$ through parameter t_1. The resulting equations for the curve $\overline{P}_{ch(2)}(t_2=f(t_1))$ take the following form in *Maple* computer algebra system:

$$x_{ch(2)}(t_1)=-2\cdot RootOf(4096\_Z^5-576t_1\_Z^4+(1024t_1^2+8192)\_Z^3+(48t_1^3+23760t_1)\_Z^2+$$
$$+(-192t_1^4-63360t_1^2+13312)\_Z+3t_1^5-48t_1^3-24384t_1)^2,$$
$$y_{ch(2)}(t_1)=3\cdot RootOf(4096Z^5-576t_1\_Z^4+(1024t_1^2+8192)\_Z^3+(48t_1^3+23760t_1)\_Z^2+$$
$$+(-192t_1^4-63360t_1^2+13312)\_Z+3t_1^5-48t_1^3-24384t_1),$$

where $0 \le t_1 \le 1$. The function *RootOf* is *Maple*-specific and indicates that the underlying expression is not solvable in radicals. However, this does not prevent the program from providing correct end results for both symbolic and numeric calculations.

As a result, every point of the curve $\overline{P}_{ch}(t_1)$ bijectively corresponds to a certain point of the curve $\overline{P}_{ch(2)}(t_2 = f(t_1))$. As follows from Figure 3, the sought points A_{1i} are the midpoints of straight line segments intersecting the curves $\overline{P}_{ch}(t_1)$ and $\overline{P}_{ch(2)}(t_2 = f(t_1))$ in points of equal values of parameter t_1. Symbolically these points can be determined by establishing a linear dependency between the correspondent points of curves $\overline{P}_{ch}(t_1)$ and $\overline{P}_{ch(2)}(t_2 = f(t_1))$:

$$x_1(t_1) = x_{ch(1)}(t_1)(1-\lambda) + x_{ch(2)}(t_1)\lambda,$$
$$y_1(t_1) = y_{ch(1)}(t_1)(1-\lambda) + y_{ch(2)}(t_1)\lambda,$$

where $0 \le t_1 \le 1$, $\lambda = 0{,}5$. The equations for orthogonal projection $\overline{P}_1(t)$ of the sought curve $\overline{P}(t)$ evaluated in *Maple* computer algebra system are of the following form:

$$x_1(t_1) = 1 - 0{,}125t_1^2 - RootOf(4096\_Z^5 - 576t_1\_Z^4 + (1024t_1^2 + 8192)\_Z^3 + (48t_1^3 + 23760t_1)\_Z^2 + \\ + (-192t_1^4 - 63360t_1^2 + 13312)\_Z + 3t_1^5 - 48t_1^3 - 24384t_1)^2,$$
$$y_1(t_1) = 4t_1 + 1{,}5RootOf(4096Z^5 - 576t_1\_Z^4 + (1024t_1^2 + 8192)\_Z^3 + (48t_1^3 + 23760t_1)\_Z^2 + \\ + (-192t_1^4 - 63360t_1^2 + 13312)\_Z + 3t_1^5 - 48t_1^3 - 24384t_1).$$

As we can see from Figure 1b, any point $A_1(x_1(t_1), y_1(t_1))$ of curve $\overline{P}_1(t)$ constitutes an orthogonal projection of a projecting cone vertex. Evidently, the continuous multitude of the vertices matches the sought spatial curve $\overline{P}(t)$. Therefore, for example, in case of *α*-projection, applicate *z* value of a point A_1 will be equal to the distance from the point A_1 to one of the curves $\overline{P}_{ch(1)}(t_1)$ or $\overline{P}_{ch(2)}(t_1)$ in normal direction with respect to curve $\overline{P}_1(t)$. Therefore, applicate $z_1(t_1)$ can be determined from the equation $z_1(t_1) = \sqrt{(x_{ch(1)}(t_1) - x_1(t_1))^2 + (y_{ch(1)}(t_1) - y_1(t_1))^2}$:

$$z_1(t_1) = (1 - 0{,}125t_1^2 + RootOf(4096\_Z^5 - 576t_1\_Z^4 + (1024t_1^2 + 8192)\_Z^3 + \\ +(48t_1^3 + 23760t_1)\_Z^2 + (-192t_1^4 - 63360t_1^2 + 13312)\_Z + 3t_1^5 - 48t_1^3 - 24384t_1)^2)^2 + \\ +(4t_1 - 1{,}5RootOf(4096\_Z^5 - 576t_1\_Z^4 + (1024t_1^2 + 8192)\_Z^3 + (48t_1^3 + 23760t_1)\_Z^2 + \\ +(-192t_1^4 - 63360t_1^2 + 13312)\_Z + 3t_1^5 - 48t_1^3 - 24384t_1))^2)^{1/2}.$$

The acquired spatial curve $\overline{P}(t) = (x_1(t), y_1(t), z_1(t))$ is the sought one. It also allows one to determine the classic cyclographic projection through the equations (1). Figure 6 depicts the final result of solution of the considered example with orthogonal projection $\overline{P}_1(t)$ of the curve $\overline{P}(t)$. Additionally, for further clarity, the figure also represents the cyclographic *α*-projections $\overline{P}_{\alpha(1)}(t)$ and $\overline{P}_{\alpha(2)}(t)$ acquired with half-angle 45° at the projecting cone vertex. Orthogonal projections of the generating ruled surfaces directed by the give curves $\overline{P}_{\alpha(1)}(t)$, and $\overline{P}_{\alpha(2)}(t)$ as well as the resultant curve $\overline{P}_{\alpha(2)}(t)$ are also depicted.

The results of numerical experiments have justified the theoretical conclusion that transition from the curves $\overline{P}_{ch(1)}(t_1)$ and $\overline{P}_{ch(2)}(t_2)$ acquired through modified cyclographic mapping back to the initial curve $\overline{P}(t)$ is possible. It may happen that there will be an analytic solution given second-degree curves $\overline{P}_{ch(1)}(t_1)$ and $\overline{P}_{ch(2)}(t_2)$ or outlines of second-degree curves. In other cases, e.g. given spline curves of third degree or higher, the spatial curve $\overline{P}_{ch}(t_1)$ can be reconstructed in the form of discrete multitude of analytically acquired points. The actual curve $\overline{P}_{ch}(t_1)$ can be then reconstructed through interpolation of this multitude.

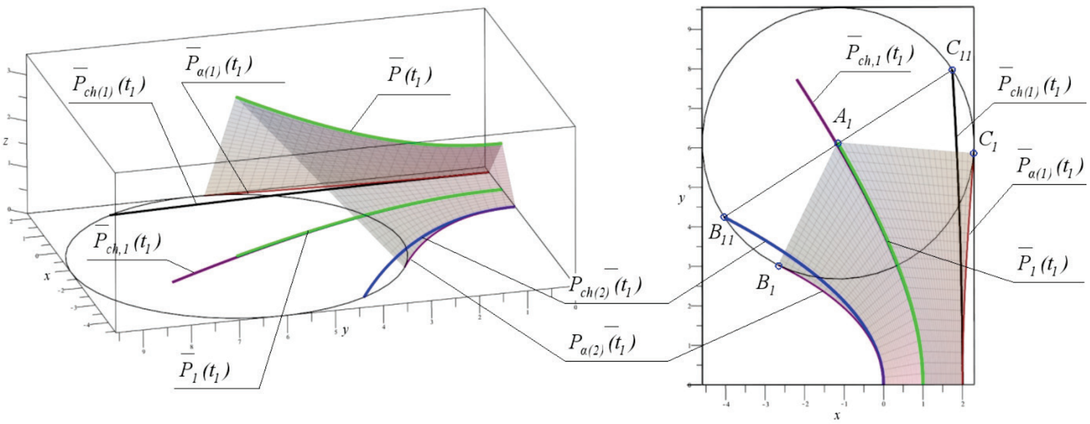

Figure 6: Visualization of the final result of reconstruction of the curve $\overline{P}(t)$ (figure shows its orthogonal projection $\overline{P}_1(t)$)

4. Conclusions

The results of the study demonstrate that modified cyclographic mapping, just as classic cyclographic mapping, is reversible. The proposed algorithm for reconstruction of the initial spatial curve given the curves acquired through modified cyclographic mapping is based on the algorithm for the inverse problem of classic cyclographic mapping of a spatial curve. It allows one to uniquely define the orthogonal projection $\overline{P}_1(t)$ of the sought curve and reconstruct the sought spatial curve $\overline{P}(t)$. The results of the study can be applied in development of computer-aided design systems for design of roads of both common and special purpose.

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