# Geometric Modeling of Parabolic Power Particle Streams

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#### Abstract

This paper is devoted to geometric simulation of parabolic stream area in planar case. The stream is characterized by fuzziness of geometric parameters that is consequence of the technological parameters fuzziness. In this paper we consider the process of welding spark flying as a physical analogue of the parabolic stream. The method relies on consideration interval sets and combinatorial computing analyses of various geometric objects in planar case. In particular, this approach is used for parabola having interval numerical parameters. Various aspects of the parabolic stream such as shadow sub-areas and dangerous zones of the stream are discussed. We break up the area of the stream into interval closed sub-areas which correspond to interval physical parameters of metal drops. We also demonstrate that the most probable dangerous parts of welding protective suit can be discovered by developed model. The practical application of the method of geometric modeling of welding sparks makes it possible to determine the localized areas of the parts of a protective suit that need additional protection in terms of improving materials and design.

#### Keywords

Conical separation, interval parabolas, parabolic flow, welding sparks, stratified area.

### 1. Introduction

Various power streams are to be considered in many branches of industry. Geometric simulation is the formal study of these processes. At present there are many problems connecting with geometric approaches to the modeling but the problem of using geometric methods in order to find effective model for particle streams is still of interest for scientists. There is great interesting class of problems connecting with the problem of radiant energy in physical fields. One can meet these problems, for example, in computer graphics, robotics, architecture, building and so on.

The stream of radiant energy is usually determined with respect to some point which is considered as a source of energy. Also, the stream is considered together with some receiver of energy. The stream in itself is simulated by central or parallel pencil of straight lines. It corresponds to a physical principle of energy spreading.

Geometric analyses of the space together with some objects and some energy stream is usually based on the following principles. Firstly, there exists the principle of discrete approach. It is a case where all objects of the space are considered approximately as sets of points. Secondly, many of the results are simpler when the methods of combinatorial computing geometry are used. All various generalizations mean, as a rule, increase in number of energy sources, variation of reflecting surface forms and also variation of receiver characteristics.

In this paper we will take a look at the other generalization of the problem. Namely, we will give up the idea of straightness for energy stream. Also, we will give up the principle of continuity for the stream. In this research we show that curved discrete streams of particles, for example, the stream of spray, dust, grains of sand or welding sparks may have geometric description.

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# 2. Object and aim of research

The aim and the main objective of the present paper are to reconstruct computable geometric model of curved discrete power streams. One of the problems is investigation and description flight paths of power particles in some physical fields. For example, it may be magnetic field or the field of gravitation. To solve the problem it is necessary to have some volume of experimental data. In order to find dangerous sections of the space or dangerous parts of some objects we can divide the space into special domains which are called strata. Each stratum corresponds to certain physical parameters of the flying particles. Moreover, it is necessary to determine spatial geometric form of the strata. Taking into account various objects and their mutual locations into the strata we would like to propose computable geometric method of analyses including determination of shadow sub-domains.

The problem arises naturally in the hand-operated arc welding process which is an increased risk process. The stream of high-temperature sparks is the main dangerous factor. The temperature of falling sparks may be about 600°C and amount of such sparks may be appraised 90 per cent.

### 3. General theoretical considerations

In this paper we take into consideration the two-dimensional Euclidean space  $P^2 = X \times Y$  and the following objects.

1. Some number of compact sets  $M_1, M_2, ..., N_1, N_2, ...$  with certain properties, like closed convex ones, or even polygons.

2. The boundaries of the sets which are approximated by broken lines  $M_1 = \{M_{11}, M_{12}, ..., M_{1m}\}, \dots, N_1 = \{N_{11}, N_{12}, ..., N_{1n}\}, \dots$ 

3. The sources of power eradiation which are simulated by points  $A, B, \ldots$ .

4. The receivers of power eradiation which may be lines *a*, *b*, ... or even curves.

5. The domain **D** of power eradiation stream.

Two sets in the plane are conically separable if there is a conic separating these sets. Let  $M = \{M_l, M_2, ..., M_m\}$ ,  $N = \{N_l, N_2, ..., N_n\}$  and let f = 0 denotes the equation of the conic. If  $f(M_i) < 0$ ,  $f(N_j) > 0$ ,  $l \le i \le m$ ,  $l \le j \le n$ , or on the contrary,  $f(M_i) > 0$ ,  $f(N_j) < 0$ , M and N are conically separated by the conic f which is called a conic separator.

Two sets in the plane are conically separable with respect to a point A if there is a conic f(A) = 0 which separates these sets. Also, two sets in the plane are conically separable with respect to points A and B if there is a conic f(A) = 0, f(B) = 0 which separates these sets. If we increase the number of points we may consider the sets which are conically separated with respect to five points in the plane.

Two sets are parabolically separable if there is a parabola  $f = ax^2 + bx + c$  which separates these sets. Parabolic separability is a special case of conic separability. Polynomial separability of the sets is a generalization of the parabolic separability. Two sets are polynomially separable if there is a polynomial  $f = a_nx^n + ... + a_2x^2 + a_1x + a_0$ , which separates these sets. It means  $f(M_i) < 0$ ,  $f(N_j) > 0$  or on the contrary  $f(M_i) > 0$ ,  $f(N_j) < 0$ . In this research we consider the parabolic polynomial separability with respect to one or two points.

All computing problems of parabolic separability may be solved by means of combinatorial analyses of supporting parabolas. Supporting parabola of a given set is defined as a parabola having at least one point common with the set and all other points of the set are on the same side of the parabola. Supporting parabola of a given polygon is defined as a parabola passing through one, two or three vertices of the polygon. Supporting parabola may be defined as a parabola having one, two or three points of contact with the given set. There are some combinations of these conditions.

We are not going to study parabolic separability of various sets. We will now take a superficial look at some obvious properties of parabolic separability.

- 1. Two linearly separable sets are parabolic separable ones.
- 2. Two sets which are linearly separable with respect to a point are parabolically separable ones with respect to the same point.
- 3. Two sets which are linearly separable with respect to two points are conically or parabolically separable ones with respect to the same points.

Figure 1 shows some examples of linear, conic and parabolic separability. The sets M and N are linearly separable sets. Moreover, they are linearly separable sets with respect to the point A. But they are not linearly separable sets with respect to the point B. One can see that M and N are conically separable sets with respect to the point B. One supporting conic which is the conic separator is shown. Also, one can see that the sets M and N are parabolically separable ones with respect to the points A and C.

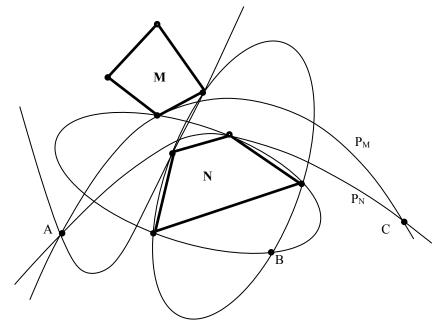


Figure 1: Some examples of linear, conic and parabolic separability

### 4. Strong and weak parabolic separation. Interval parabolas

Let  $p_M$  and  $p_N$  denote two supporting parabolas of several sets in the plane. The sets are separated by an infinite family of parabolas if  $p_M$  and  $p_N$  generate this family.

Two sets are strongly parabolically separable if  $p_M$  and  $p_N$  are not coincided parabolas and if they generate some one-parametric set of parabolas. This one-parametric set of parabolas is called a strong parabolic separator. It there exists only one separating parabola two given sets are weakly parabolically separable. This parabola is called a weak parabolic separator of the sets.

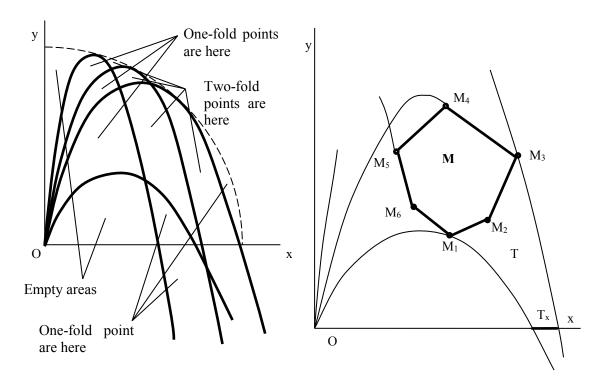
There are two theoretical considerations of the strong parabolic separator. Firstly, one can interprets the strong parabolic separator as an infinite continuous set of parabolas. For example, it may be a part of bi-central pencil of parabolas filling the domain between  $p_M$  and  $p_N$ . Secondly, one can consider the strong parabolic separator as an interval parabola, i. e. the parabola having one interval parameter. For example, such parabola may be defined by the following interval equations

$$y = [a_{min,} a_{max}]x^{2} + bx + c, y = ax^{2} + [b_{min,} b_{max}]x + c, \text{ or}$$
  
$$y = x^{2} + bx + [c_{min,} c_{max}].$$

Some continuous stream of light, liquid or gas between some objects in the space may be considered as a physical analogue of strong continuous separability. But some streams such as the stream of spray, dust, grains of sand or welding sparks between some objects in the space may be simulated by interval sets of trajectories.

Let us consider one-central pencil of parabolas having the point O(0, 0) as the centre. Assume that  $y = [a_{min,}a_{max}]x^2 + [b_{min,}b_{max}]x$ , (1)

for real interval numbers  $[a_{min}, a_{max}]$  and  $[b_{min}, b_{max}]$  is the equation of interval parabola in the plane. The interval parabola divides some neighborhood of the point O into several parts. Some parts have not any points of the parabola, some parts have one-fold points and the other parts have two-fold points (see Figure 2).



**Figure 2**: Sub-domains generated by interval parabola (on left side) and shady areas generated by the set *M* (on right side)

## 5. The area of welding spark flying

Metal splashes are the result of hydrodynamic blow during some welding process. Visually, splashing is represented by flying of welding sparks. In order to investigate welding sparks it is necessary to take into account the range of flying, diameter of the drop, temperature of the drop, mass of the drop, time of cooling, velocity of flying and others.

Based on experimental data we construct geometric model of area not far of welding arc or welding torch. One can see that some parameters are interval scalars and the other parameters are interval vectors. It is clear that generally a flight path of the spark can be determined by interval parabola. Based on the presented considerations we can conclude that the geometric model of the area has fuzzy-interval and vector-scalar structure in the field of gravitation.

Let  $t_0$  be the initial moment of time. Let the metal drop has a mass *m*, constant thermal capacity *Q* and initial temperature  $T_0$ . Let the temperature of environment be a constant denoted by  $T_C$ . Let us assume the temperature of the drop is falling from  $T_0$  to  $T_K$ .

The velocity of drop cooling is proportional to the difference of the temperatures, i. e.

$$mQ \cdot dT/dt = -k(T - T_C).$$
<sup>(2)</sup>

Hence,

$$T - T_C = C e^{-kt/mQ}.$$
 (3)

From this we have

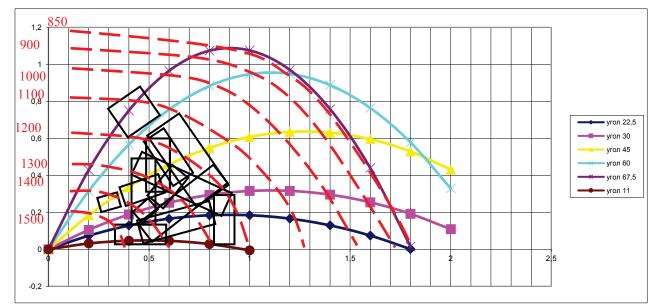
$$T = T_C + (T_0 - T_C)[(T_K - T_C)/(T_0 - T_C)]^s, s = t / t_K.$$
(4)

The trajectory of spark flying is a parabola if the air resistance and all other factors are ignored. The parabola has equation

$$y = tg\alpha_0 \cdot x - g / (2V_0^2 \cdot Cos^2\alpha_0) \cdot x^2.$$
<sup>(5)</sup>

Here  $\alpha_0$  is an initial angle of the trajectory and  $V_0$  is an initial velocity of the sparks. One can easy to compute the flying time from  $t_0$  to the meeting with the x-axis and some other parameters such as the distance from the source up to the point of meeting with the x-axis and the height of the trajectory. For example, the time of flying up to the point having abscissa x is

$$t = x / (V_0 \cdot Cos \alpha_0). \tag{6}$$



Such a model is shown in Figure 3 and Figure 4.

Figure 3: The strata of welding sparks flying when  $V_0 = 5$  m/s,  $T_0 = 1600^{\circ}$ C,  $T_c = 20^{\circ}$ C,  $\alpha = [11, 67.5]$ 

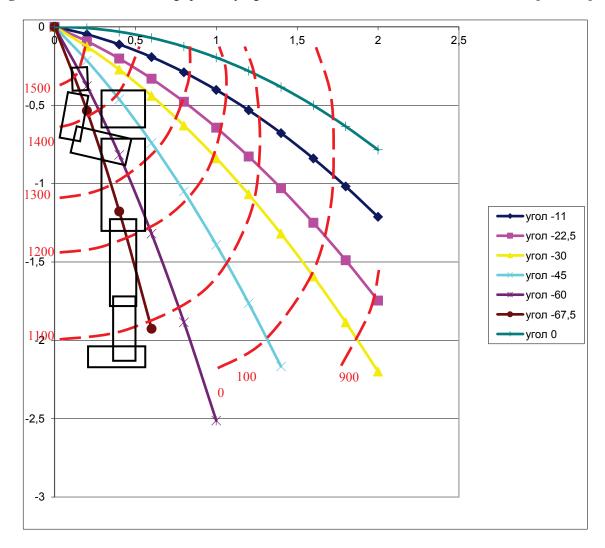


Figure 4: The strata of welding sparks flying when  $V_0 = 5$  m/s,  $T_0 = 1600^{\circ}$ C,  $T_C = 20^{\circ}$ C,  $\alpha = [11, 67.5]$ 

Based on experimental data we can use the interval velocity  $V_0 = [4, 8]$  m/s. Then we are able to construct geometric model of the area of spark flying. The area consists of strata which are bounded by closed convex interval surfaces corresponding to temperature intervals. Such a model is shown in Fig. 3 and Fig. 4.

# 6. Application of the model to designing of protecting clothes

In this section we will take a general look at application of the model. The model may be applied in simulation and designing some processes connecting with energy particles flying. Welding process is characterized by high risk for respiratory ducts, skin and eyes. There is the whole complex of individual protecting means such as masks, gloves, footwear and especially protective closes. Experimental data shows that any protective clothes are exposed to damage more than all other means. Because of their physical properties protective clothes are liable to burning. Various fire-resistant impregnations are not always effective.

When studying the protective properties of special clothes one has to have in mind the existence of dangerous zones in the space of technological process. Determination of dangerous zones is connected with rather difficulties which are usually results of technological indefiniteness of various physical parameters. Therefore, the model is based on interval temperature strata in the space not far from the source. Using the model we are able to determine the degree of thermal coercion to all objects located into the strata.

Taking into account such properties of fabrics as fire-resistance we can compute the time of suit destruction by molten drops. Fig. 3 and Fig. 4 show the dangerous zones of protecting suit for various poses of welder. One can see that arms, forepart of thigh and chest are the most dangerous parts in parabolic particle stream. The other parts of protecting suit are in shadow area.

The basic idea of this paper can also be applied to other branches of industry. For example, it may be of great significance in metallurgy, petroleum gas and chemical industries. There exist a possibility of using the model for forecasting various results of explosion and other processes connecting with the flying of particles.

# 7. Conclusions

We have proposed the fuzzy-interval and vector-scalar geometric model of welding area in order to describe flying of welding sparks. In this paper we have presented some general properties of interval parabolic trajectories of sparks. Throughout this paper we have considered the planar case only. We think that the fuzzy-interval and vector-scalar geometric model needs further investigation. One of directions in which the theory could be generalized is to investigate the space case.

Using the investigation we can forecast the most dangerous zones and save zones into the area. Also we are able to determine dangerous zones of protecting suit for various poses of welder.

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