Modeling and Visualization of Lava Flows

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Abstract

The study of the behavior of lava flows plays an important role in predicting, preventing and reducing the consequences of volcanic eruptions. Lava has been used as a building material for centuries and has been a source of nutrients for agriculture, but lava flows remain a threat to human activities.

The model of spreading of a viscous inhomogeneous incompressible fluid under the action of gravitational forces is used to describe the lava flow process. The mathematical model is described by the Navier-Stokes equation and the continuity equation with the corresponding initial and boundary conditions. The model takes into account the variable viscosity of the lava, which depends on the volume fraction of crystals.

As a spreading surface, we use the generated topography, which is a realistic slope of a mountainous area, formed taking into account natural geological processes. Numerical simulation is carried out using the meshless SPH method. The results of several model cases of lava flows over the surface are presented.

Simulation results are visualized using our custom-developed Cinema Science 3D approach. It allows a custom 3D visualization to be programmed using simple CSV file. We used it for presenting our results in a natural view, showing underlying terrain as mesh and lava as points, moving and changing according to time and other computation parameters. This view was enough for achieving visualization aims of our research.

Keywords

lava, numerical simulation, scientific visualization, smoothed-particle hydrodynamics, SPH.

1. Introduction

Numerical simulation of viscous fluid flows is being widely used in studies of volcanic processes and plays an important role in understanding the dynamics, morphology, and thermal evolution of lava flows [1]. In effusive eruptions, a lava flow begins to form when molten rock erupts onto Earth's surface and slowly spreads across the surface from a volcanic vent. Volcanic eruptions produce a variety of lava flows depending on the chemical composition and temperature of the erupting material, and the topography of the surface over which the lava flows [2, 3].

Thermal effects play an important role in lava flow, but simplified isothermal analytical and numerical models have demonstrated how lava flows in the absence of cooling [3, 4]. In this

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paper, we continue to study the processes of crystallization and formation of lava domes [5], so we will not take into account thermal effects.

In the framework of this work, the simulation of the flow of a viscous fluid by the meshless methods of Smooth Particle Hydrodynamics [6] is considered, as well as the problem of visualizing the results.

2. Problem formulation

Mathematical model The model of spreading of a viscous inhomogeneous incompressible fluid under the action of gravitational forces will be used to describe the process of lava flow. The mathematical model describing such a motion is given by the Navier-Stokes equation and the continuity equation, which in Lagrangian coordinates have the form

$$\rho \frac{D\mathbf{u}}{Dt} - div \left(\mu \left(grad \, \mathbf{u} + \left(grad \, \mathbf{u} \right)^T \right) \right) = -grad \, p + \rho \mathbf{g},\tag{1}$$

$$\frac{D\rho}{Dt} = -\rho \, div \, \mathbf{u} = 0, \tag{2}$$

where

- $\mathbf{u} = (u_1, u_2, u_3)$ velocity vector,
- $\mathbf{g} = (0, 0, -9.81)$ free-fall acceleration vector,
- *p* pressure,
- ρ density,
- μ dynamic viscosity,
- *t* time,
- grad and div are gradient and divergence respectively.

As a spreading surface, we use the generated topography, which is a realistic slope of a mountainous area, formed taking into account natural geological processes.

A volcanic crater with a diameter of 30 meters has been formed on this surface, in which a particle emitter is located, which generates particles with a constant initial velocity $\mathbf{u} = \mathbf{u}_c$. On the rest of the surface *G*, the no-slip condition $\mathbf{u} = 0$ is specified.

The initial parameters of a particle leaving a crater have the following values, shown in the 1 table. These parameters correspond to the average values of eruptions with basaltic lavas.

Table 1

Initial parameters of a particle emerging from a crater

Parameter	Value	
Density	$\rho = 2600$	$kg \cdot m^{-3}$
Initial speed	$u_{c} = 0.01$	$m \cdot s^{-1}$

We assume that the viscosity of lava depends on the volume fraction of crystals [7, 8]

$$\mu(\varphi) = \mu_0 \left(1 + \varphi^{\delta} \right) \left[1 - (1 - \xi) \cdot \operatorname{erf} \left(\frac{\sqrt{\pi}}{2(1 - \xi)} \varphi(1 + \varphi^{\gamma}) \right) \right]^{-B\phi_*},$$
(3)

where $\varphi = \phi/\phi_*$, ϕ is the volume fraction of crystals and ϕ_* – specific value of the volume fraction of crystals; *B* is the theoretical value of the Einstein coefficient (it has been experimentally established that the Einstein coefficient ranges from 1.5 to 5 [9]); $\delta = 7.24$, $\gamma = 50.76$ and $\xi = 4.63 \times 10^{-4}$; erf(·) is the error function.

The volume fraction of crystals is determined from the evolution equation describing the simplified kinetics of crystal growth during crystallization due to magma degassing

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \mathbf{u}) = -\frac{\phi - \phi_{eq}}{\tau},\tag{4}$$

with initial condition $\phi(t = 0, \mathbf{x}) = 0$. Here ϕ_{eq} is the volume fraction of crystals in equilibrium, which depends on the amount of water dissolved in the magma and on temperature; τ is the crystal growth time. The smaller τ , the faster the crystallization process converges to its equilibrium state.

Surface topography Several test scenes were generated to simulate lava flow. One of the scenes represents a section of a mountain surface created in the Wold Machine package [10]. This scene has sections of steep and gentle descents for expert assessment of the correctness of the flow behavior under various conditions, as well as formed active and inactive craters. Hydro- and wind- erosion parameters characteristic of Hawaiian-type volcanoes were used for formation. An example of such a surface is shown in figure 1.



Figure 1: Artificial lava flow surface, generated taking into account the features inherent in Hawaiiantype volcanoes

3. Numerical Simulation Results

Computational experiments were carried out for the lava viscosity $\mu_0 \in \{10^2, 10^3, 10^4, 10^5\}$ on the time interval $[0; 2.5 * 10^6]$. Figures (2) show the results of numerical simulation.



(c) $\mu_0 = 10^4$

(d) $\mu_0 = 10^5$

Figure 2: The figure shows the dynamics of lava spreading with dynamic viscosity depending on the level of crystals in the melt according to the formula (3) at the same time

4. Visualization of results

In addition to the numerical calculations themselves, there was also the task of visualizing the results of the simulation. This task is relevant, since the visual method for evaluating the results of numerical modeling is very common in volcanology.

The following aims were set for visualization:

- Show the dynamics of lava spreading over time;
- Within one visualization, show the results of several series of calculations with different initial parameters;
- Show the internal structure of the lava flow.

The first aim is of primary interest to an expert volcanologist, since it allows one to qualitatively assess the dynamics of spreading and the morphology of the resulting dome, and thus assess the overall "quality" of the simulation. The second aim is born from the frequent impossibility of specifying the exact values of the initial lava parameters. According to indirect data, a volcanologist can indicate parameters in a certain range. Therefore, a frequent request is the variation of some of the modeling parameters in one of the directions. The third aim is more "cosmetic", as many rendering packages allow to slice object with a plane. To assess the dynamics of the distribution of some parameter (for example, the level of crystals or the distribution of relict particles) in space, a larger number of such slices may be required.

A CinemaScience 3D approach [11], developed at the Krasovskii Institute was chosen as the visualization tool. It allows a custom 3D visualization to be programmed using simple CSV file. In this file, which describes a table in a text format, three things are defined:

- 1. a list of parameters,
- 2. a list of visual artefacts, including their types,
- 3. a mapping between parameters values and artefact files.

Both parameters and visual artefacts are abstract to CinemaScience approach. The following model is used: when user selects some parameters values in graphical interface, a corresponding visual artefact files are loaded and presented in 3D space. Thus a wide range of visualizations may be programmed, including time-based.

To solve the first of two visualization aims defined above, the paradigm of describing the results of calculations in the CinemaScience format is perfect, since it allows to represent each variable condition as a visualization parameter.

Thus, the figure 2 shows different initial calculation parameters (variation of the initial viscosity parameter) for a fixed time parameter. On the contrary, the figure 3 shows the spreading dynamics at different moments of time for a fixed parameter μ_0 .

The last aim was achieved by using sets of "modifiers" ("extras")– relatively simple scripts that allow to modify the behavior of the scene. The 4 figure shows the Clip serie modifier, which allow to make a regular set of sections parallel to the plane and with a given thickness. This makes it possible to evaluate the distribution of parameters within the flow over the entire volume simultaneously.

Summarizing, using the CinemaScience 3D approach for scientific visualization, we got the following results:

Firstly, thanks to the simple description of the scene in CSV format, we are able to display several computational results corresponding to the same model time in one scene. This means that both static time slices and animated sequences can be displayed. This possibility is very important for retrospective and comparative analysis. So, we can specify the calculation parameters as a visualization parameter and view the progress of the simulation process for different computational processes at one point in time.

Secondly, the description of the scene is not static and may change during the visualization process. This makes it possible to build the complex of online visualization. When new numerical results appear, new entries are added to the scene description. This allows the researcher to monitor the progress of the simulation and, if necessary, make changes to the model parameters or stop the calculations without waiting for the end of the numerical simulation. This is important because it can significantly save computing time. Another advantage of online visualization is the ability to process results directly from the supercomputer's storage, without the need to move large amounts of data.





(c) 25 hours;

(d) 36 hours;

Figure 3: The dynamics of lava spreading with dynamic viscosity depending on the level of crystals in the melt according to the formula (3) with viscosity $\mu_0 = 10^3$ at different times



Figure 4: The figure shows a lava flow sliced with the Clip serie modifier. The color indicates the fraction of crystals, from 0.6 (yellow) to 0.8 (black). (*a*) clearly shows the formation of a "channel" of the flow, during which the lava adhering to the surface thickens and forms a trough. (*b*) shows how a solid crust is formed and a relatively liquid flow underneath, which can lead to the formation of lava "tubes" or cracking of the crust. (*c*) shows the formation of a lava dome with hard crust, which is formed during the outflow of highly viscous lava

And thirdly, the use of the modifiers system allows you to quickly create the necessary types of displays, including filters and additional entities that allow you to more fully explore the visual object. So, the Clip serie modifier described above allows you to literally cut a point cloud into plates of the required thickness and with a convenient step, literally with one click,

which allows you to see the internal structure of the flow. In comparison, in ParaView [12], this would require creating modifiers equal to the number of cuts and manually modifying them (which makes it very heavy and unstable).

5. Conclusion

A three-dimensional model of lava flow evolution was considered under the assumption that the lava viscosity depends only on the volume fraction of crystals, and this fraction, in turn, depends on the characteristic time of crystal growth. Lava flows are modeled using the SPH method. The visualization of the numerical results was performed using CinemaScience 3D approach. Using the visualization tool compatible with that approach, various slices of lava flows were examined. The distribution and formation of crystals were analyzed depending on the initial viscosity. This information is used for short-term and long-term forecasts of the eruption of real volcanoes by employees of volcanological observatories.

It is also necessary to add that, although the above results were obtained on synthetic data, they are as close as possible to real data. At the moment, numerical simulations are being carried out on real topologies obtained by satellite scanning.

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A. Online Resources

You can take a look at the ViewZavr project (which is an implementation of the described approach) and example scenes via

https://github.com/viewzavr/vr-cinema