Comparison of BSDF Reconstruction Methods for Rough Surfaces

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Abstract
The work is devoted to the problem of reconstruction of a bi-directional scattering distribution functions (BSDF) for rough surfaces. The paper contains short overview of different methods and comparison of their results. The elements with rough surfaces are widely used in modern optical devices, for example, such as light guiding plates for illuminating system of displays, car dashboards, or luminaires. So, accurate and effective reconstruction of scattering properties of rough surfaces is important for visualizations tasks and generation of photorealistic images. Typically scattering properties of rough surface are described with help of BSDF. In some cases, BSDF can be just measured; however in many cases direct measurement is not sufficient. For example, if it is required to define BSDF inside of the material. Such measurement is impossible or too expensive because neither detector of the measuring device nor the light source can be placed inside the material. Therefore, a lot of different approaches to reconstruct BSDF for rough surfaces were developed. The approaches are based on both wave and ray optics and use different analytical and numerical solutions. A great variety of approaches results in the situation when you would like to know what method could be selected as more appropriate for the specific sample. This work investigates a bunch of perspective methods of BSDF reconstruction for surfaces of different roughness. The verification is based on numerical as well as visual comparison with real measured data. Finally, the general conclusions and recommendations are presented about what method and for what applications is more appropriate.

Keywords
Rough Surface, Bi-Directional Scattering Function, BSDF, Surface Scattering, Cook-Torrance model, GGX model, Rendering, Visualization, Wave Optics, Ray Optics.

1. Introduction

The definition of scattering properties for a boundary between two media is a simple task if the boundary is smooth. In such a case the light transmission and refraction can be easily simulated using Snell's law of refraction and reflection. However, in the case the boundary is rough the definition of light propagation through such surface is a complicated task. As a rule, scattering properties of a rough surface are specified with Bi-Directional Scattering Distribution Function (BSDF) which determines output angular light distribution (refraction and reflection) depending on input light conditions.

In most trivial cases, when only input/output angular light transformation through the rough surface is important the direct BSDF measurement may be sufficient. The ordinary way of BSDF measurement for the rough surface is presented on the Figure 1a. A sample, one side of which is rough, is illuminated with a parallel light beam under the specific incident directions, then an angular light distribution of transmitted light (BTDF - Bi-Directional Transmittance Distribution Function) and reflected light (BRDF – Bi-Directional Reflectance Distribution Function) is measured. In other words, such BSDF measured for the whole sample can be used in cases when ignoring of sample thickness is permissible. The examples of such ordinary BSDF applications may be different diffuse films, layers (Figure 1b).

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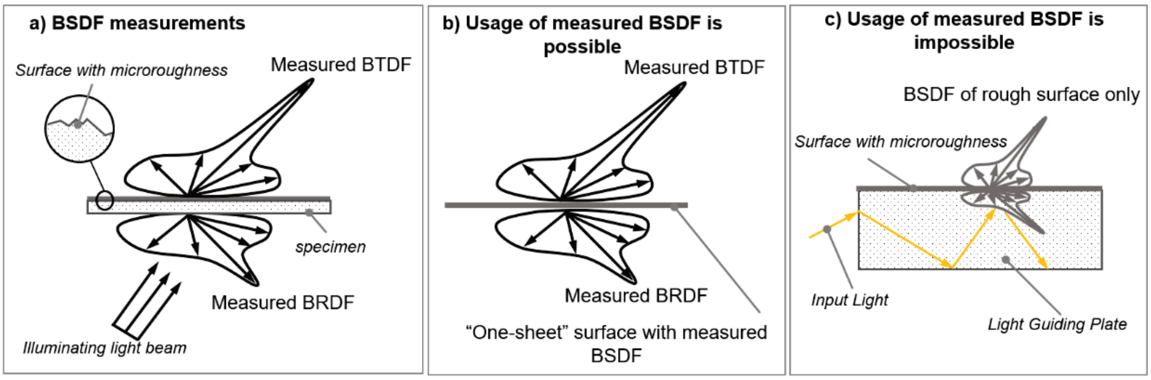

Figure 1: BSDF application in the simplified "one-sheet" and the more complex "solid" models

However, there are a lot of cases where the direct usage of BSDF measured for "one-sheet" surface is impossible, for example, in light-guiding plates with rough surfaces, see Figure 1c. To simulate light propagations correctly in such system it is required BSDF from each side of the rough surface that includes BSDF from the material side. The measurements of such BSDF are impossible or very expensive, because it is a problem to place the light source and detector inside of the material. Moreover there are multiple interreflections from both sample sides which should be excluded during BSDF calculation. Another problem is the significant inaccuracy of BSDF measurement for the grazing illumination angles because of light leakage inside of measured samples, shadowing of sample illumination, and some other reasons.

The mentioned problems related to BSDF measurement result in the development of a lot of different approaches to BSDF reconstruction. One of the main purposes of this paper is an analysis and verification of the well-known methods of BSDF reconstruction. A comparison is done for more prominent approaches on the base of real measured samples with rough surfaces.

2. BSDF reconstruction methods used in the comparison

Generally, all methods of BSDF reconstruction can be divided into the two main approaches:

1. **Methods use measurement of heights distribution** for rough surface with profilometers or microscopes. The description of the approach can be found in [1]. The approach is based on measurement of height distribution of rough surface. The data allow to build real geometry of rough surface (for example, using "microfacet" model). Then knowing optical properties of sample material (refractive index, absorption) it is possible to build the computer model of whole rough sample. The surface properties typically are specified as pure Fresnel supposing all surfaces of the microfacet model are smooth (polished). The next step in the approach is illumination of the sample with parallel light under different incident light directions and the calculation of light reflected and transmitted with rough surface. This scattered light finally defined BSDF. In the case of computer virtual model there is no problem to place light detectors and sources inside of the sample material. The approach based on the measured high distribution for rough surface can be subdivided into two branches depending on what optics, wave or ray, is used for light simulation.
   1.1 **Methods based on ray optics.** In the case of ray optics typically simple Forward Monte-Carlo ray tracing is used to calculate light scattering. The ray methods are simple and reliable, but they have limitations of ray optics and can result in inaccuracy of BSDF reconstruction because wave effects are ignored, especially it can be noticeable for surfaces with small roughness. Another problem of ray methods is their high sensitivity to high distribution values. Parameters of height distribution measurement like step, noise can result in noticeable difference in the shape of reconstructed BSDF.
   1.2 **Methods based on wave optics.** The main problem of wave methods is their extremal complexity. The precise wave methods cannot be applied practically due to the complexity of micro-surface geometry. So very approximate wave solutions are mainly used for BSDF reconstruction. The most well-known and widely used BSDF reconstruction method is based

on the Kirchhoff approximation. The approach is built on a simple FFT (Fast Fourier Transform) procedure. A more detailed explanation can be found in [2-7]. The Kirchhoff approximation wave approach is developed for both reflection and transmission components and will be examined below. The method does not support complex light transformation on rough surface profile like masking-shadowing or interreflections. Thus the less roughness of surface the better accuracy will be achieved with the Kirchhoff approximation.

Both methods based on measured profile data are used in the comparison. An optimization procedure was run for both methods to achieve better quality of BSDF reconstruction. The angular transmittance measured for all rough samples was used as an optimization goal. The measured height distribution is modified (scaled, smoothed or filtered) during optimization to achieve the better agreement between measured and simulated angular transmission.

2. **Methods do not use measurement of heights distribution**. In general the measurement of height distribution of rough surfaces is expensive. Moreover sometimes these data cannot guarantee the accurate BSDF reconstruction. So in the absence of measured height distribution another group of methods can be applied. They use parametric presentation of roughness, where the parameters of rough surface are defined during optimization procedures. There methods can be subdivided in two groups: analytical and numerical.

2.1 **Analytical methods.** The analytical methods are based on physical, optical theory, empirical formulas and “microfacet” model of rough surface. A lot of methods have been invented like “Ward”, “Ward-Duer”, “Blin-Phong”, “Cook-Torrance”, “Lafortune et al”, “He et all”, “Ashikhmin-Shirley”, “GGX” etc. [8-32]. The main advantage of analytical methods is their high efficiency because analytical solutions are fast to calculate. This is an important feature because typically optimization processes are used to define required parameters of analytical functions describing required BSDF shape. The disadvantage of the approaches is that the approximate algorithms are used to describe complex optical effects such as masking-shadowing of the incident light illuminating of the rough surface. This can introduce inaccuracy during BSDF reconstruction which is noticeable for surfaces with big roughness. In this paper the “GGX” model is selected for investigation. It is an improved variant of the Cook-Torrance [9] microfacet model. It supports as a reflection as well refraction and shadowing-masking. The paper [27] contains numerical data comparing different analytical models and demonstrates their advantages relatively to other analytical methods of BSDF reconstruction of rough surface. The “GGX” model is considered as the most accurate, flexible, and widely used. It supports both reflection and refraction components, masking-shadowing and importance sampling. It shows more accurate output than the Cook-Torrance model according to [27].

2.2 **Numerical methods.** Nowadays with increasing of computer power new methods of BSDF reconstruction have been developed in [1, 33, 34]. Part of them is pure numerical ones. Numerical methods are based on building a geometry model of rough surface and calculation of BSDF by light simulation. The method based on the normal density distribution of rough surfaces is proposed in [1, 33] and on the height distribution – in [34]. Here a micro-relief is built with help of normal or height distribution represented by an analytical function which parameters are defined with help of the optimization process. The method based on the normal distribution is simpler in calculation, has faster convergence during optimization but, however, has restrictions. It ignores interreflections on facets of rough surface and does not support masking-shadowing also. The method based on the height distribution is more complex, the rough surface geometry is reconstructed here. Thus it supports all complex light interactions on microfacets. Its drawbacks are complexity and worse convergence during optimization. In any case both methods show good results in BSDF reconstruction according to [33, 34] and are used in comparison in our paper. In principle both, ray and wave, optics can be applied to calculate light scattering in the numerical methods. However we use only ray optics approach because it shows better results in most of cases according to our experiments.

Finally, the following methods are used in the comparison:

1. **Measured (ray optics)**. The approach uses measured height distribution and ray optics to calculate light scattering.
2. **Measured (wave optics).** The approach uses measured height distribution and wave optics to calculate light scattering.

3. **Analytical (“GGX”).** The analytical approach is an advanced variant of the Cook-Torrance model.
4. **Numerical (“Normals”).** The numerical approach uses “Normals” distribution.
5. **Numerical (“Heights”).** The numerical approach uses “Heights” distribution.

3. Samples of rough surface

Before describing the sample to be used in the investigation let’s consider a profile rough surface (Figure 2).

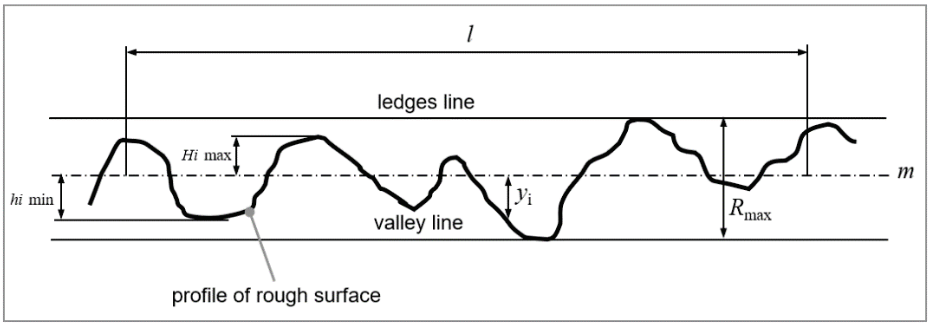

Figure 2: Parameters of micro-roughness

Several widely used parameters describe a profile of rough surface. These parameters will be used for the description of measured samples. So we shortly consider them. The first parameter Ra is an arithmetical mean deviation of the assessed profile. It is the most common and is calculated with the formula:

$$R_a = \frac{1}{n}\sum_{i=1}^{n}|y_i| \tag{1}$$

The next parameter R_q is a root mean square:

$$R_q = \sqrt{\frac{1}{n}\sum_{i=1}^{n} y_i^2} \tag{2}$$

The eight samples made of acryl with refractive index = 1.49 are selected for investigation. Each sample has one rough side and another is smooth. Two types of measurements are fulfilled for all samples:

1. A height distribution was measured with the precise Taylor Hobson profilometer.
2. Angular distribution of scattered light was measured with goniophotometer GP-200 by urakami Color Research Laboratory (Figure 3).

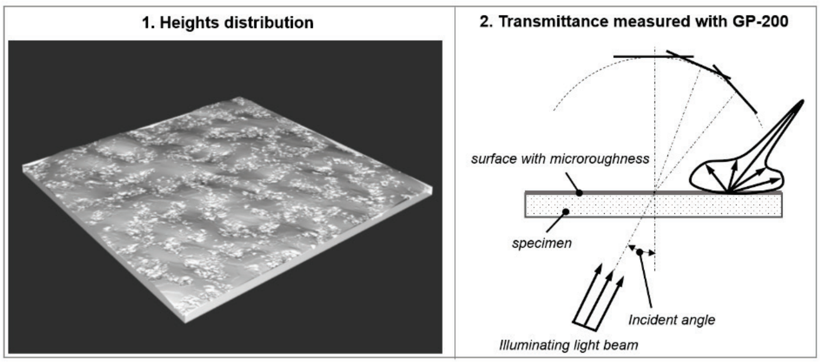

Figure 3: Input measured data of investigated samples

The parameters of all eight profiles have been calculated based on measured height distributions and using formulas (1) and (2). These parameters are combined in Table 1. The #1-#8 in the first column are sample identifiers. Additionally, the second and third rows of the Table 1 present size of the measured fragment and resolution of measurement – number of measured profile points along both x and y directions. The step between measurement points was constant.

Table 1
Profile parameters of measured samples

Param./Samp.	#1	#2	#3	#4	#5	#6	#7	#8
Size (mm x mm)	0.37x0.37	0.95x0.95	0.95x0.95	0.95x0.95	0.37x0.37	0.95x0.95	0.95x0.95	0.95x0.95
Resolution	1024x1024	1333x1333	1330x1330	1332x1332	1024x1024	1332x1332	1333x1333	667x677
Ra (μm)	0.178	0.456	0.668	0.738	1.170	2.038	2.596	10.724
Rq (μm)	0.232	0.581	0.866	0.956	1.466	2.669	3.308	13.456

The images of investigated profiles are presented on Figure 4.

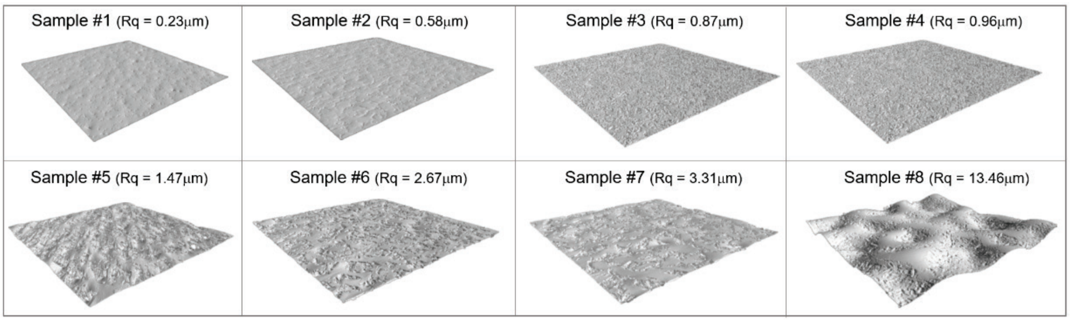

Figure 4: The appearance of measured profiles

For convenience profiles on Figure 4 are placed in order of increasing of their root mean square (Rq) from the left to the right and from up to down.

The GP-200 goniophotometer [36, 37] was selected for measurements because it has very advanced characteristics, such as angular resolution = 0.6°, very small angular step = 0.1° and wide range of observation directions = ±90°. The high angular resolution is very important because some of measured samples have very small roughness, comparable with wavelength. So it is supposed that the angular distribution of scattered light has a very narrow shape. The measurement of transmitted light was done for five incident light angles = 0°, 15°, 30°, 45° and 60°in the single plane of light incidence. The goniometer GP-200 outputs data in the relative values calibrated to measurement without sample. So, for the correlation of measured and simulated data the same optimization process is fulfilled in simulation as it is explained in [37].

4. Comparison of different BSDF reconstruction methods

The verification of all selected methods was fulfilled in the Lumicept programming complex [35]. The software has special instruments for direct light simulation of rough microgeometry on the base of numerical height and normal distribution. Also it has physically accurate Monte-Carlo ray tracing and BSDF generators allowing to calculate BSDF based on ray as well as on wave optics (Kirchhoff approximation). For verification of GGX analytical approach a special plugin was developed and integrated into Lumicept software.

The reconstructed BSDFs for all samples and constructed by all methods were used in the simulation of angular distribution of transmitted light for the model similar to real measurement scheme of GP-200 goniophotometer explained in the previous chapter.

However, such numerical comparison can be insufficient. BSDF of rough surface can have complex shape and even small inaccuracy in its generation can result in the image defects, appearance of some artifacts. Especially it can be noticeable if BSDF is attached to complex curved objects which are illuminated under grazing angles. So, it is preferable to check how BSDF samples look under some realistic conditions. For this a special virtual scene aimed at BSDF sample visualization was prepared (Figure 5). The scene represents a special measuring box JUDGE-II by X-Rite [38]. It is a box with surfaces close to diffuse and set of luminescent tube lamps emulating daylight. The several test objects: a plate, a sphere, and a torus are placed into the measuring box. The reconstructed BSDF is attached to the external surface of the test objects. Internal surfaces of test objects are simulated as ideally smooth and have perfect Fresnel properties. The medium of all objects has the refractive index = 1.49 which corresponds to measured samples.

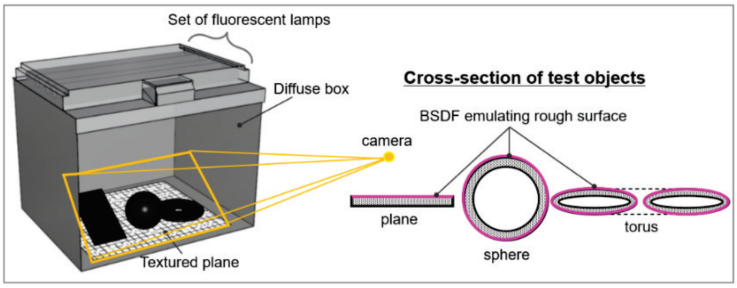

Figure 5: Scheme of scene for visualization

The scene is observed from a finite distance point with a sensor emulating the human eye or camera. The image is generated with help of the forward Monte Carlo ray tracing in Lumicept [35]. It is not the most effective tool nowadays from viewpoint of efficiency and calculation speed and the generated images, as a rule, contain noise. However it is the most reliable tool because of its simplicity.

5. Results

The results of the simulation are presented in the two variants:

1. As graphs with intensity of transmitted light. A special scene to simulate this characteristic as precisely as possible has been prepared. It is maximally close to the measurement scheme of GP-200 goniophotometer [37]. The simulation was done for five incident directions of parallel light in one plane corresponding to the plane of light incidence. The incident light angles δ = 0°, 15°, 30°, 45° and 60°.
2. All images were generated according to explanations in Chapter 4.

The simulation was fulfilled for five methods of BSDF reconstruction explained in Chapter 2. Figures 6-13 present result in the same format. The first row presents graphs of angular distribution of transmitted light in the ordinary linear scale. The next row contains similar graphs of angular distribution in the logarithmical scale.

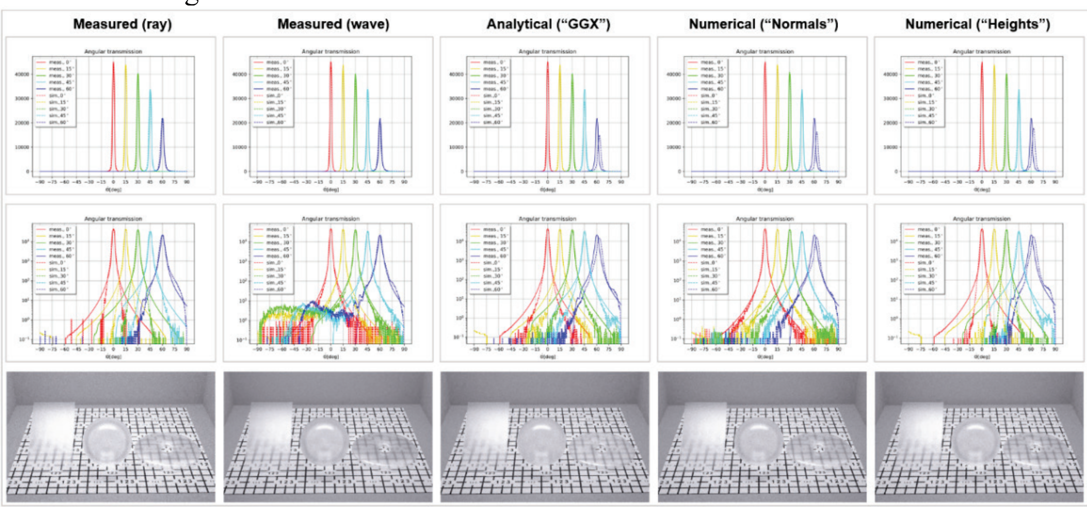

Figure 6: Results for sample #1 (Rq = 0.23μm)

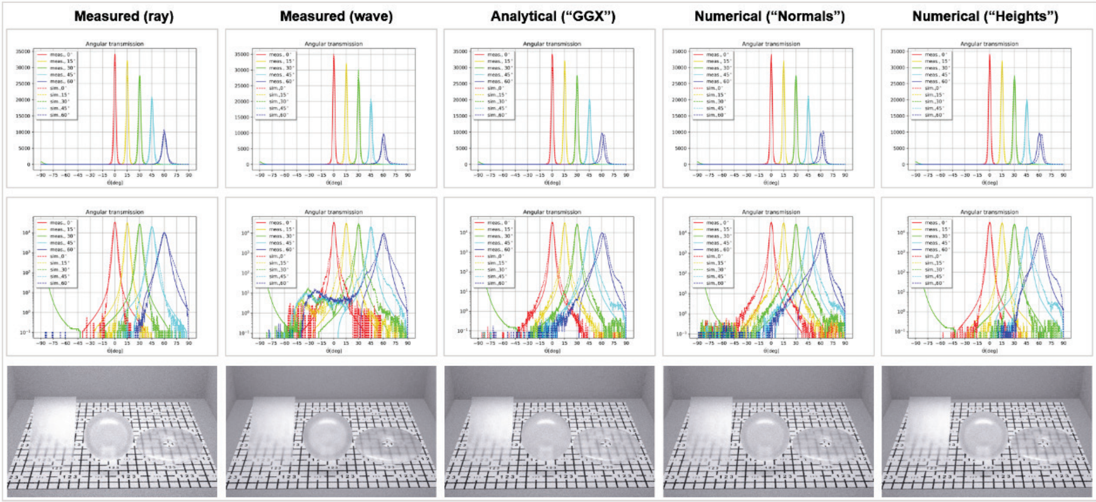

Figure 7: Results for sample #2 (Rq = 0.58μm)

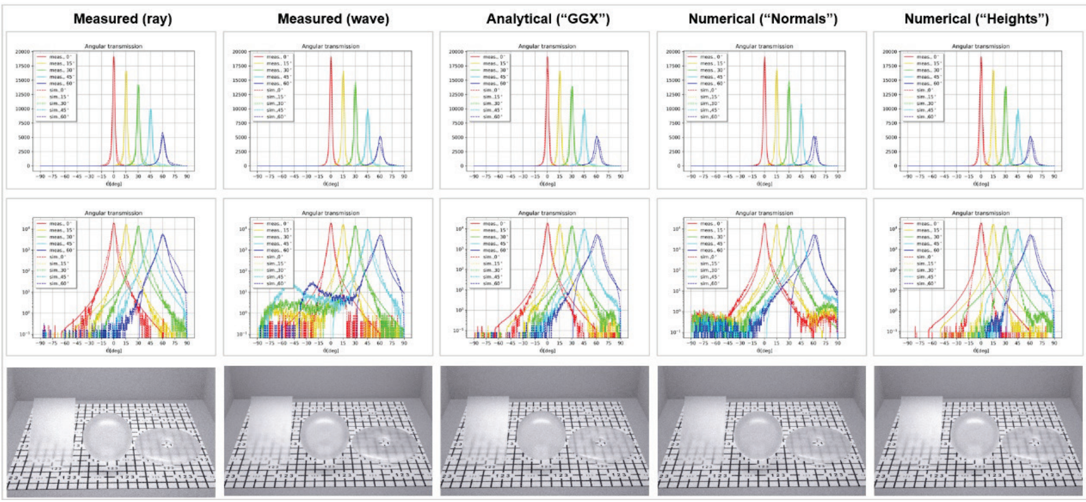

Figure 8: Results for sample #3 (Rq = 0.87μm)

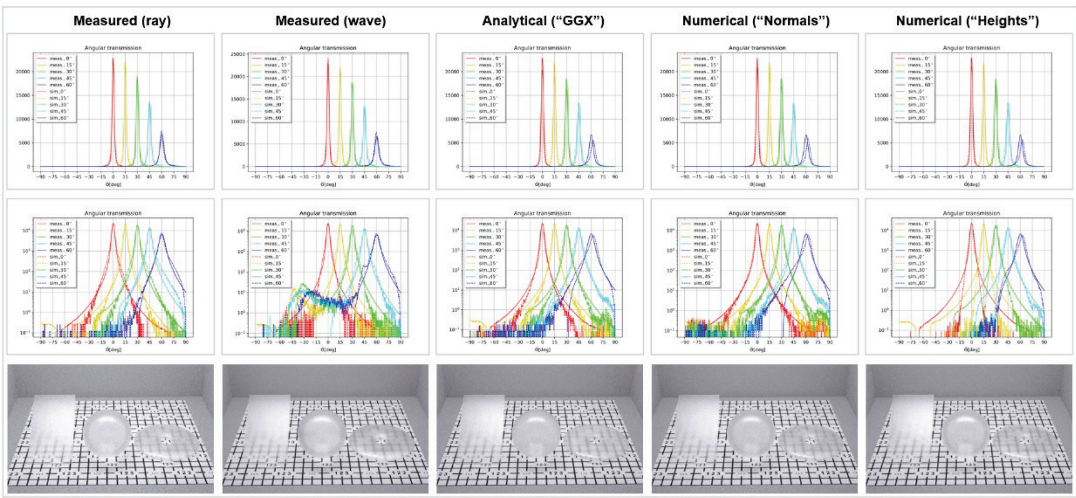

Figure 9: Results for sample #4(Rq = 0.96μm)

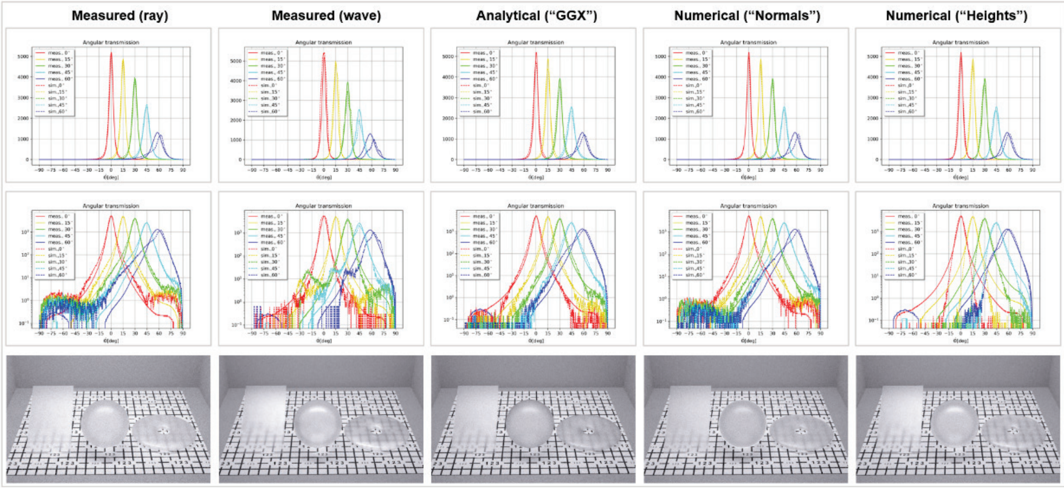

Figure 10: Results for sample #5 (Rq = 1.47μm)

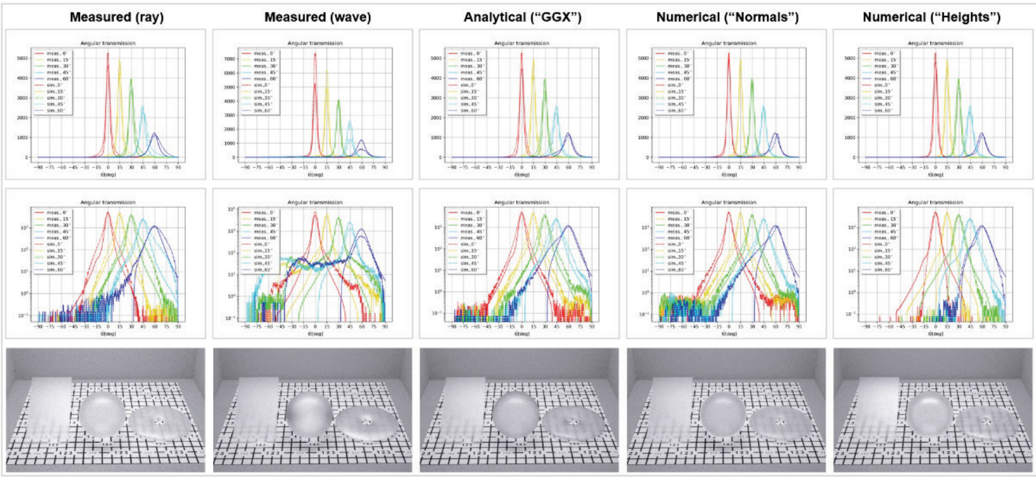

Figure 11: Results for sample #6 (Rq = 2.67μm)

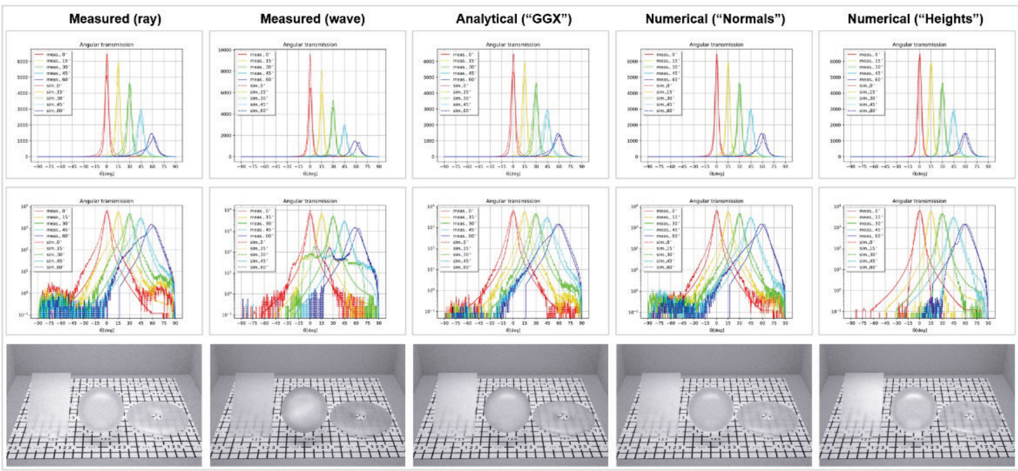

Figure 12: Results for sample #7 (Rq = 3.31μm)

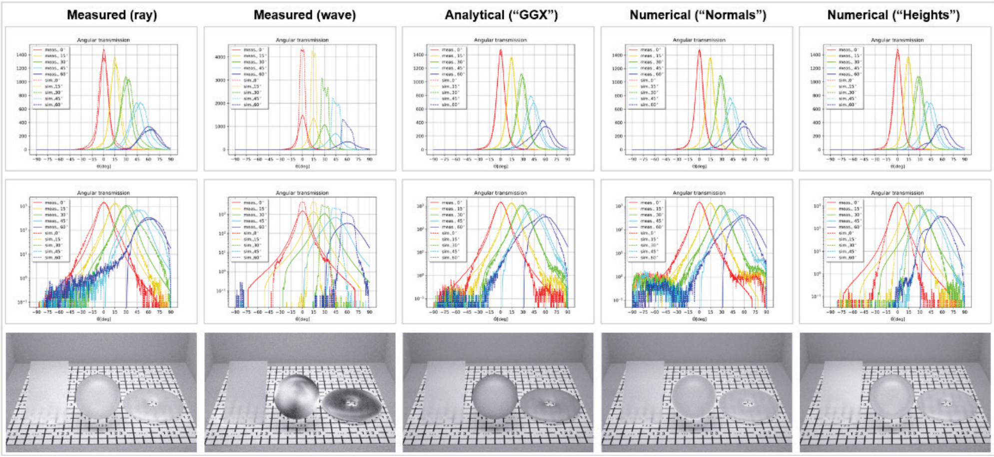

Figure 13: Results for sample #8 (Rq = 13.48μm)

6. Conclusions

Numerical results of BSDF reconstruction, angular distribution of transmitted light in the graphs of the Figures 6-13, demonstrate that most of the examined methods work quite well. The exception is “Measured (wave)” method based on the Kirchhoff approximation where we see a noticeable difference for samples #5-#8 with Rq > 1μm. It is quite predictable according to the restriction of the Kirchhoff approximation. So, the wave approach cannot be recommended for usage for samples with big roughness (Rq > 1μm). On the other hand the graphs for samples #1-#3 demonstrate that the wave approach gives more correct result for the angular distribution shape. Moreover wave approach does not practically require any optimization of measured height distribution unlike ray optics methods. It can be explained with the big sensitivity of the ray approach to the quality of microrelief measurement (noise, step between measured nodes). The analytical “GGX” method (the improved Cook-Torrance model) works reasonably. For the “GGX” method a noticeable inaccuracy appears only for samples with big roughness. So, the method can be recommended for simulation of rough surfaces with average roughness. It should be noted that the “GGX” method is simple because only one parameter manages BSDF shape.

Comparing methods based on measured height distribution (“Measured (ray)” and “Measured (wave)”) versus all other methods for samples with small roughness, it should be pointed out that agreement between measured and simulated intensity, especially for grazing illumination angles, is better for methods which use measured geometry of roughness. In the same time these methods often require optimization of the geometry. Both numerical methods show good agreement with measurement practically for all examined samples. The numerical “Normals” method is slightly better in the area of general transmission estimation than the “Heights” method, has better convergence during optimization, and is simpler in the calculation.

Some conclusions can be done from the images rendered using reconstructed BSDF. From physical theory we can say that the methods based on additional height distribution measured data are supposed to be more precise because the real profile geometry is used during ray transformation. In the case of “Measured (ray)” method the interreflections, shading, and masking effects are supported in the whole volume because a full-functional Monte Carlo ray tracing is used. The errors here can be caused by the restrictions of ray optics, inaccuracy of measurement of height distribution or the measured fragments are not representative. However, they can be overcome with modification of microrelief during optimization procedure (at least partially). Thus the “Measured (ray)” method can be considered as a reference (“etalon”) in visual comparison with other methods. The plates in all images are similar to each other. The rendering of the curved object is more complicated. The images generated with the analytical “GGX” approach are similar to the reference image in case of small and average roughness. However, if to consider small details like a dark ring on the sphere, the effect is absent in any examined samples created with help of the “GGX” method. And the curved objects appear darker for samples with big roughness. Likely the energy conservation works not so well for approximate analytical methods for samples with noticeable roughness. The numerical “Normals” method generates quite good images for the #1-#4 samples but images for rougher samples have noticeable artifacts like bright edge ring on the sphere. The reason for this effect is evident because the method does not support interreflections, masking, and shadowing. From the viewpoint of visual appearance the numerical “Height” method shows the best results for all samples.

Summarizing all simulated data if precise simulation is required and measurement of microrelief geometry is not available then the numerical “Heights” distribution method is more accurate. In case of small roughness the “GGX” or “Normals” methods can be sufficient.

7. Acknowledgments

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8. References

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