

Computer analysis and restoration of motion blurred images

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Histogram based and correlation based algorithms for computer analysis and restoration of motion blurred images have been proposed. The histogram algorithm uses ridge detection method and direction distribution histogram analysis. The correlation algorithm finds a detection function, that when superimposed on the Fourier spectrum of the motion blurred image, gives a minimum. The results of comparison between the histogram method and the correlation method are presented. The tests show practically good results for the realistic motion blur lengths.

Keywords: image blur, motion bur, blur kernel estimation, ridge detection, histogram method.

1. Introduction

Motion blur on the image occurs when the relative motion between the scene and/or the sensor occurs during exposure. Such an effect makes the recognition and perception of images difficult. To apply algorithms to restore blurred images, it is required to evaluate the blur kernel. Thus, success of blurred image restoration significantly depends on an accuracy of blur parameters estimation.

This paper is the continuation of [1], where an algorithm for image blur kernel estimation for the case of uniform motion blur was proposed. In this work the proposed histogram method was adapted for the blur kernel, which uses a third, additional, Gaussian blur parameter. A comparison between the histogram method and correlation method based on the idea suggested in [4] has been also performed.

For motion blur parameter estimation various techniques are being used. A method based on the analysis of power cepstrum of the image has been proposed in [2]. In [3] motion blur estimation is based on the analysis of shape, the homogeneity and the smoothness. It is proposed to analyze the correlation between the Fourier spectrum of blurred image and a periodic detecting function in [4], and the results of comparison between method [4], [2] and [3] were presented. In [5] a spectral analysis of image gradients had been performed to identify the blurring kernel, in [6] it was proposed to analyze the Hough transform. In [7] the authors suggest using modifications of the Radon transform for the identification of the blur spectrum pattern. In [8] a novel PSF estimation scheme based on frequency spectrum analysis has been proposed.

The motion blur parameter estimation is used in various areas. As an example, in [9] motion blurred images analysis was used for vehicle speed detection. Also, motion blur estimation is used for document image deblurring [10].

Formulation of the problem. In this paper we consider the uniform linear motion blur. Blurred image can be represented as:

$$g(x, y) = f(x, y) * h(x, y),$$

where $h(x, y)$ - is a blur function, or PSF, $f(x, y)$ - uncorrupted image, $g(x, y)$ - blurred image.

The PSF (Point Spread Function), or blur kernel, in the case of uniform blur, can be defined as [11]:

$$h(x, y) = \begin{cases} \frac{1}{L}, & \sqrt{x^2 + y^2} \leq \frac{L}{2}, y = x \tan(\theta) \\ 0, & \text{else} \end{cases}, \quad (1)$$

where L, θ are the length and the angle, which defines blur direction, respectively. Figure 1 shows an example of the PSF (1).



Figure 1. An illustration of the PSF function (1), corresponded to $\alpha = 45, L = 15$). The contrast of the image was increased.

Also, we used the Gaussian function for motion blur simulation:

$$h(x', y', \alpha) = e^{-\left(\frac{x'^2}{2\sigma_x^2} + \frac{y'^2}{2\sigma_y^2}\right)}, \quad (2)$$

where

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}. \quad (3)$$

In this model we use 3 parameters: $\sigma_x, \sigma_y, \alpha$ - length, width and direction respectively. Adding a width parameter and a Gaussian blur makes the model more appropriate to real images.

Thus, in this paper the definition of motion blur parameters is the determination of length and direction or length, direction and width. The ridge detection method [12] and direction distribution histogram analysis are used as the main techniques.

This paper is organized as follows. Section 2 includes the histogram algorithm description. Correlation method for motion blur parameters estimation is described in Section 3. The testing results and their discussion are given in Section 4.

2. Histogram algorithm for motion blur parameters estimation

Algorithm for determining the direction of the motion blur vector. To determine blur direction we use an approach based on ridge detection.

The ridges (valleys) of a smooth function of two variables are a set of curves whose points are local maxima (minima) of the function in at least one dimension.

The curves whose points correspond to the maxima are called ridges, and whose points correspond to the minima are called valleys. We use ridges for motion blur analysis.

Let us consider an image as a function of two variables $I(x, y)$. For each pixel we compose the Hessian:

$$\begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}.$$

Let λ_1 and λ_2 be the eigenvalues of this matrix, then ridges can be defined as $\lambda_{max} = MAX(\lambda_1, \lambda_2)$, where the maximum denotes the eigenvalue with the greatest magnitude and where $\lambda_1 \gg \lambda_2$. The direction of the eigenvector, which corresponds to λ_{max} define direction of the ridge.

To extract the skeletons corresponded to ridges we apply non-maximum suppression algorithm. This step significantly increases the accuracy of determining blur direction.

Thus, for each point of the selected ridge we can define the angle of inclination of the tangent to the ridge by following formula:

$$\theta = \tan^{-1}(I_{xy}/(\lambda_{max} - I_{xx})). \quad (4)$$

When the motion blur occurs, the contours of the image are stretched in the corresponding direction, which we take as the desired parameter. To determine it, we construct a direction distribution histogram. Note that the angle θ is rounded to integer value when computing histogram.

The horizontal axis in the histogram corresponds to the value of the angle from 0° to 180° , and the vertical axis shows the number of pixels with the corresponding angle value calculated from the formula (4). To increase the accuracy, we use for histogram construction not only the pixels selected by non-maximum suppression algorithm, but also their 1 pixel radius neighborhood. We take the average maximum obtained from the peak analysis on the histogram as the desired direction parameter.

Algorithm for determining the length of the motion blur vector.

To determine the length of the motion blur vector, we suggest gradually blurring the image in a direction that is perpendicular to the existing blur direction, gradually increasing the blur length. The initial length of the perpendicular blur and the step inside the loop can be chosen depending on the considering task. With each iteration the image becomes

more and more defocused, and the direction distribution histogram changes.

We denote the value $S_L = \sum_{i=0}^{90} hist(i)$ ($S_R = \sum_{i=90}^{180} hist(i)$) as the left sum (the right sum) of the direction distribution histogram.

Thus, with each iteration the histogram tends to the moment when the sum of the left part of the histogram is approximately equal to the sum of the right part of the histogram. It is proposed to leave the loop at the moment when the corresponding sum of the histogram becomes greater than the opposite sum. We take the value of the iteration parameter, at which the sign of the difference between the left and right histogram sums has changed, as desired parameter of length.

Algorithm for determining the "width" of the motion blur kernel.

The ridge detection method depends on the parameter σ . Changing it we can find ridges at different scales. Therefore, for different values of σ , we can get different values of the direction of the blur vector.

Thus, the accuracy of determining the direction of motion blur depends on the accuracy of finding the parameter σ , which corresponds to the width of the motion blur kernel (2). Finding a good approximation of σ makes possible to increase the method accuracy and makes the method automatic. For these purposes we used a multiscale ridge detection method.

We build a grid of the following values $\sigma \in \{\sigma_1, \dots, \sigma_n\}$, $\sigma_i = \sigma_0 \nu^{i-1}$, where $\sigma_0 = 1$, $\nu = \sqrt{2}$. We used $n = 6$. Consequently, the corresponding values are equal to $\sigma = \{1, \sqrt{2}, 2, 2\sqrt{2}, 4, 4\sqrt{2}\}$. For each σ_i we perform ridge detection and get the image $H^{\sigma_i}(x, y)$. To equalize the values between Hessians, obtained for different σ_i , we multiply the derivatives of the image by σ^2 at the construction stage. To suppress false filter responses we use threshold value of 0.1 for small σ .

It is assumed that if there are point sources on the image, than the number of pixels on the skeletons corresponded to ridges will predominate in some direction. Basing on this consideration we choose σ_i corresponding to the image with the largest number of ridges.

3. Correlation method for motion blur parameters estimation

To compare the results obtained by method described at section 2, we developed a correlation algorithm based on the idea from [4]. Such approach can be applied to the images corrupted with motion blur, which corresponds to (1) model. Let us denote correlation algorithm as "algorithm T".

The main idea of the algorithm T is to choose such a detection function, which, when superimposed on

the Fourier spectrum of the motion blurred image, gives a minimum.

We consider a blurry image $g(x, y) = h(x, y) * f(x, y)$, where $h(x, y)$ is PSF, $f(x, y)$ is uncorrupted image and $g(x, y)$ is blurred image. Then the Fourier transform of image (hereinafter referred to as FT) can be represented as $G(\xi, \eta) = H(\xi, \eta)F(\xi, \eta)$, where $G(\xi, \eta), H(\xi, \eta), F(\xi, \eta)$ - are FT of $g(x, y), h(x, y), f(x, y)$ respectively. The main idea of the method is to represent the function $H(\xi, \eta)$ using *sinc*-function. On images that have a blur kernel close to the model (1) we can observe zero-patterns on Fourier spectrum. It is required to identify zero-patterns on the Fourier spectrum of the blurred image by estimating the correlation between the detection function $\rho(\mu)$ and the Fourier spectrum of the blurred image.

We assume that the detection function $\rho(\mu)$ is periodic. Consequently, changing the period we can achieve the smallest value of the correlation function.

The correlation function based minimization method can be presented as

$$I(L, \theta) = \int \int w(\xi, \eta) \rho(\mu(\xi, \eta; L, \theta)) \log |G(\xi, \eta)| d\xi d\eta,$$

$$I(L, \theta) \rightarrow \min,$$

where $w(\xi, \eta)$ - weighting function, $\mu(\xi, \eta; L, \theta) = L(\xi \cos \theta + \eta \sin \theta)$. To reduce the influence of noise, the weight function is built according to the rule: the higher the frequency, the less weight.

The detection function was built according to the following rule:

$$k(\mu) = \left| \frac{\sin(\mu)}{\mu} \right| * 500 - \frac{1}{100 * G(x, y, \sigma_x, \sigma_y)},$$

$$\mu = \frac{1}{l}(\xi \cos \theta + \eta \sin \theta),$$

$$z(\mu) = \begin{cases} \frac{k(\mu)}{2}, & |\mu| < \frac{l}{2} \\ 0, & z(\mu) < 0, \\ k(\mu) & else \end{cases},$$

$$\rho(\mu) = \begin{cases} 255, & z(\mu) > 0, |z(\mu)| < 10 \\ 0, & else \end{cases},$$

where $G(x, y, \sigma_x, \sigma_y)$ - Gaussian function, $\sigma_x = 30, \sigma_y = 70$. The parameter α corresponds to the direction of the blur, and l corresponds to the blur length. To perform the rotation of this function, the coordinates are transformed using the formula (3). Figure 2 shows an example of the detection function and the Fourier spectrum of the blurred image.

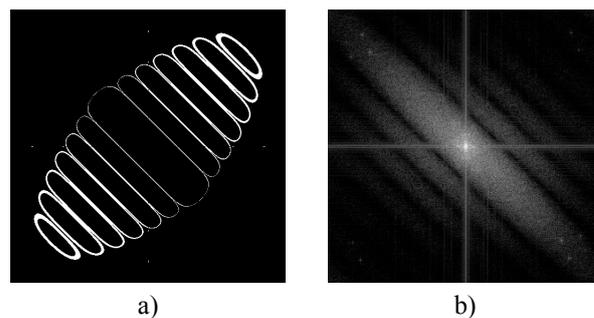


Figure 2. Detection function (a) corresponded to Fourier spectrum of baboon image ($\alpha = 45, L = 5$) (b).

4. Testing and results

Comparison of the results of histogram and correlation algorithms. The comparison results (just an example) of the histogram algorithm and the correlation algorithm are given in the Table. 1. Note that here we use half of the length.

$L/2-\phi$	$L/2$	$L/2(\text{method T})$	ϕ	$\phi(\text{method T})$
5-45	5	6	43°	42°
7-45	7	6	43.5°	42°
8-45	7	11	43.5°	45°
9-45	8	11	44°	42°
10-45	8	11	44.5°	45°

Table 1. Comparison of the results with the correlation algorithm. Test images were blurred with the kernel (1).

It can be concluded that the histogram algorithm is more accurate for a small value of the parameter L than the correlation algorithm. With the increase of the blur length, both algorithms give approximately the same accuracy.

The results of the histogram algorithm. The results of the histogram algorithm applied to images blurred with a kernel (2) presented in Table.2.

$L-\alpha-W$	$\alpha(\text{estimated})$	$L(\text{estimated})$	$W(\text{estimated})$
4-30-1	4	29.5°	1.4
5-30-3	5	30.5°	2
7-30-3	7	29.5°	2
8-30-3	8	29.5°	2
10-30-4	9	28°	4
15-45-3	15	45.5°	4

Table 2. The results of the histogram algorithm applied to images blurred with a kernel (2).

Restoration of motion blurred images. Using the proposed algorithms it is possible to determine the desired parameters of the PSF, and now it becomes possible to apply an inverse regularizing method to find an unblurred image. Since this task is ill-posed [13],

even knowing the blur kernel it is difficult to restore the original image. To restore the blurred images we used a total variation based method [14]. After applying the restoration algorithm, the sharpness of the images is increased (Fig.3, Fig.4).



Figure 3. Blurred image baboon ($\alpha = 45, L = 8, W = 3$); (a); image restored using total variation deblurring [14] (b).

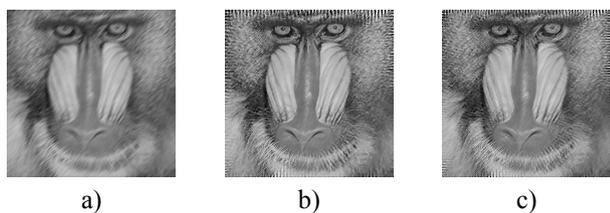


Figure 4. Image blurred with $\alpha = 45, L = 7$ (a); image (a) restored using parameters acquired using histogram method (b) and correlation method (c).

5. Conclusion

In this paper, histogram and correlation method for analyzing and reconstructing blurred images have been developed, the tests show practically good results. The histogram algorithm is more accurate for a small value of the parameter L than the correlation algorithm. With the increase of the blur length, both algorithms give approximately the same accuracy.

6. Gratitude

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7. References

- [1] *Melnikova A.A., Krylov A.S.* "Image motion blur kernel estimation using ridge analysis." GraphiCon 2017: proceedings of the 27th GraphiCon international conference, 2017. - P. 279–283.
- [2] *Cannon M.* "Blind deconvolution of spatially invariant image blurs with phase." IEEE Transactions on Acoustics, Speech, and Signal Processing 24, no. 1 (1976): 58-63.
- [3] *Yitzhaky Y., Kopeika N. S.* "Identification of blur parameters from motion blurred images." Graphical

models and image processing 59, no. 5 (1997): 310-320.

- [4] *Tanaka M., Yoneji K., Okutomi M.* "Motion blur parameter identification from a linearly blurred image." In Consumer Electronics, 2007. ICCE 2007. Digest of Technical Papers. International Conference on, pp. 1-2. IEEE, 2007.
- [5] *Ji H., Liu C.* "Motion blur identification from image gradients." In Computer Vision and Pattern Recognition, 2008. CVPR 2008. IEEE Conference on, pp. 1-8. IEEE, 2008.
- [6] *Phansalkar N.* "Determination of linear motion point spread function using hough transform for image restoration." In Computational Intelligence and Computing Research (ICCIC), 2010 IEEE International Conference on, pp. 1-4. IEEE, 2010.
- [7] *Oliveira J. P., Figueiredo M. A. T., Bioucas-Dias J. M.* "Parametric blur estimation for blind restoration of natural images: Linear motion and out-of-focus." IEEE Transactions on Image Processing 23, no. 1 (2014): 466-477.
- [8] *Wang Z., Yao Z., Wang Q.* "Improved scheme of estimating motion blur parameters for image restoration." Digital Signal Processing 65 (2017): 11-18.
- [9] *Lin H. Y., Li K. J.* "Motion blur removal and its application to vehicle speed detection." In ICIP, pp. 3407-3410. 2004.
- [10] *Qi X. Y., Zhang L., Tan C. L.* "Motion deblurring for optical character recognition." In Document Analysis and Recognition, 2005. Proceedings. Eighth International Conference on, pp. 389-393. IEEE, 2005.
- [11] *Aizenberg I., Paliy D.V., Zurada J.M. and Astola J.T.* "Blur identification by multilayer neural network based on multivalued neurons." IEEE Transactions on Neural Networks 19, no. 5 (2008): 883-898.
- [12] *Serrat J., López A., Lloret D.* "On ridges and valleys [image registration]." In Pattern Recognition, 2000. Proceedings. 15th International Conference on, vol. 4, pp. 59-66. IEEE, 2000.
- [13] *Tikhonov A.N. and Arsenin V.Y.* Methods for solving ill-posed problems. John Wiley and Sons, Inc, 1977.
- [14] *Beck A., Teboulle M.* "Fast gradient-based algorithms for constrained total variation image denoising and deblurring problems." IEEE Transactions on Image Processing 18, no. 11 (2009): 2419-2434.