Ray Tracing in Presence OF Birefringent Media*

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The paper presents the algorithm of ray transformation on a boundary of a uniaxial crystal with another crystal or an isotropic medium. It can be used in ray tracing operating Stokes formalism. The case when rays of different types have the same or not distinctive direction and thus have "mixed" polarization state is included into consideration. This method has been successfully applied to simulation of a plain light emitter with anisotropic elements sealed in its light guiding plate.

Keywords: *ray tracing, plain light emitter, birefringence, uniaxial crystals, polarization.*

1. Introduction

Modern optical engineering utilizes materials with anisotropic refractive properties. For example, in LCDs where light propagation is controlled through changing polarization state it is rather natural to include anisotropic elements before or after the liquid crystal layer attempting to improve uniformity of illumination. The approach presented here was originally developed for of device of that type, see Section 7.

Here we mainly operate a quantitative simulation of radiometric characteristics. Another application is realistic imaging, e.g., visualization of gemstones [9]. While polarization effects are usually secondary for them, other anisotropic effects such as ray splitting and "double refraction" are very important.

Both application areas (calculation of radiometric properties and realistic rendering) are rather common in Computer Graphics and utilize ray tracing, usually stochastic forward MCRT for the former and deterministic backward RT for the latter, in case of isotropic materials. Modern ray tracers support polarization. It is thus natural to add anisotropic refraction to the common ray tracing procedure. Roughly this means that we operate only ray direction and its polarization state without keeping or knowing its wave vector or direction of field etc.

2. Relation to previous work

Light propagation in anisotropic media (crystals) and its interaction with a boundary between two media, anisotropic or isotropic, had been investigated in classical optics long ago and all base relations had been obtained since Fresnel and Stokes times [1, 2]. These relations however operate wave optics parameters: the input and output are wave vector of plane wave(s) and complex amplitudes of fields in them. Whether or not polarization state (e.g. as a Stokes vector) and ray direction are enough and which form the above relations takes in these framework is not obvious. So there were a number of approaches to this problem, which can be called "polarized ray optics in crystals". A prominent publication [3, 4] had been then continued by more recent studies such as [5, 6]. In spite the problem addressed was essentially the same, different authors adapted it to different areas developing approaches optimal for their applications. The presented publication is yet another step in this direction. It deals with uniaxial (birefringent) media. It is most similar to [5] and [6], differing from [3, 4] in that no iterative calculations are used. Unlike [5] we include light incidence from anisotropic material thus being close to [6] but still operate Stokes formalism.

2.1 Rays of mixed types (not linearly polarized) in crystals

Also we eliminate the assumption common in the previous works that only "pure" state exists i.e. inside a crystal a ray corresponds to either ordinary or extraordinary wave and so has pure linear polarization. In fact this is not necessarily so and it is possible that in crystal a ray has a mixed polarization because it contains both ordinary and extraordinary waves whose rays coincide. Although this occurs under rather special conditions, they can emerge in the modern optical design. For example, if a crystal is illuminated with two waves of different polarization so that the first of them refracts into an ordinary and the second into an extraordinary waves such that their rays coincide.

A "mixed" ray inside a crystal usually reflects and refracts into distinct "pure" rays, but this, again, is not always. It is possible that its ordinary and extraordinary components reflect (or refract) into rays which have the same or very close directions. Meanwhile in any finite optical system a "parallel" light is in fact a slightly diverging beam, so it is meaningless to treat rays whose directions differ below threshold (e.g. 1 angular second) as different; they must be "merged". Then again a mixed state emerges. Our approach, like [5] whose extension it is, also operates ray direction and Stokes vector and thus was naturally included into a ray tracer originally developed for isotropic case [7]. It has been then

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applied to simulation of a plane light emitting device containing a usual glass plate with crystal tubes sealed in it. Description of the simulation and its main results are in Section 7.

2.2 Stokes parameters in crystals

Definition of Stokes vector inside anisotropic medium has certain ambiguity. In an isotropic medium one can equivalently define it either through quadratic averages of electric field or through energy flow after passing polarizers. In a crystal these definitions are not equivalent because the energy flow is a quadratic average of the field multiplied by wave number which is different for the two field directions (one is for an ordinary and another for an extraordinary components which have different wave number).

The definition through energy flow after passing polarizers is also not trivial inside crystals where polarizers work differently than in air. We (like in [5]) operate the definition through quadratic averages of electric field because it is simpler. Meanwhile it is difficult to imagine how one can measure Stokes vector inside a crystal. So the match with simulation can be checked only when the ray leaves a crystal. But there the definition through quadratic field averages is unambiguous (equivalent to others).

3. Coordinate system and notations

The boundary between two media is the plane z = 0; the normal is n = (0, 0, 1) while the incident ray has negative z-component. Optical axis is vector a (can be different in the two media).

The angle between a and the wave vector k is β; between a and the ray vector s is α.

The Stokes ort e is chosen to be electric field direction in the ordinary ray $e = a \times s$. If for the incident ray it was different then we rotate to this position also transforming the Stokes vector.

Subscripts “o” and “e” relate to the ordinary and extraordinary waves.

Refraction index is η (ordinary η_o, extraordinary η_e). Wavelength is λ. Ordinary wave number is $k_o \equiv \frac{2\pi}{\lambda}\eta_o$ (for extraordinary there is no single value but it depends on direction) and $\rho \equiv \eta_o/\eta_e$.

4. The general processing pipeline

Notice that some steps involve wave vectors, but they are used as intermediate temporary values during interaction with the boundary and forgot after that so ray tracer is unaware of them.

Input = direction of ray and its Stokes vector.

1. Calculate directions of the scattered rays; there can be many of them.

a. Calculate wave vectors of the ordinary and extraordinary waves whose ray direction coincides with the given.

b. For each of the ordinary and extraordinary incident waves, separately, calculate wave vectors of the reflected and refracted ordinary and extraordinary waves. This gives up to (some can be effectively absent) 8 scattered waves.

c. Calculate directions of rays for these outgoing waves. It is rare but possible that that several of the waves have very close rays. They are then “merged” into one ray, see (2).

2. For each outgoing ray s, calculate its Stokes vector

a. Calculate Fresnel coefficients (ratio of the field amplitude in outgoing to incident wave) for each of the 8 possible incident-outgoing combination from 2b.

b. Compose from them the elements of Mueller matrix (that relates incident and scattered Stokes vectors), then multiply the incident Stokes vector by it, see (6).

3. Forget all wave parameters, field directions etc. Only directions of up to 8 outgoing rays and their Stokes vectors remain.

5. Step1: Directions of the scattered rays

5.1 Mixed ray

In a birefringent medium, a given direction can correspond to an ordinary or to an extraordinary ray which have orthogonal linear polarizations [1], [2]. But because of the linearity of wave equation, a ray can also be a mixture of the two types.

Imagine that a crystal is illuminated (from air) with two incident beams. Each refracts into an ordinary and extraordinary rays. Now let us tune direction of the second incident beam so that its refracted extraordinary ray has the same direction as the ordinary ray created by the first incident beam. Light propagating in that direction will then be a mixture of the ordinary and extraordinary states and mixed polarization. If the incident beams are coherent, this refracted ray is elliptically polarized.

5.2 Scattering of the ordinary and of the extraordinary incident rays

So, our incident light is generally a mixture (=sum) of two incident waves, coherent or not: ordinary and extraordinary, which have the same ray direction. The outgoing light is therefore the sum of the waves scattered from the ordinary incident wave and that from the extraordinary incident wave.

For the “pure” incident waves scattering is well known [1], [2]. There are generally 2 refracted waves, one ordinary and one extraordinary; and 2 reflected waves, again one ordinary and one extraordinary. Some of these 4 outgoing waves can be evanescent (not propagating) then do not have outgoing ray.

Directions of the corresponding 4 rays are also calculated rather easily.

1. For the incident ray calculate the wave vector k from ray direction s. That of the ordinary component it is given by (7) and that for the extraordinary by

(10).
2. For each of these two:
a. Choose the local coordinate system so that Oz = normal (directed towards the incident ray) and Ox is along the tangent component of k_{incid}.
b. Calculate ordinary reflected and refracted wave vector from the incident one according to (11) and extraordinary reflected and refracted wave vector according to (13). Notice that refraction indices $\eta_o, \eta_e, \rho \equiv \eta_o/\eta_e$ are for the medium where the given scattered ray goes into.
Calculate ray directions for these 4 waves according to (14), (15)
Eventually we end with up to 8 outgoing rays. Their directions are denoted as

$$s_{o\mapsto o}, s_{o\mapsto e}, s_{e\mapsto o}, s_{e\mapsto e};$$

the first letter in the subscript relates to the type of incident ray and the second letter to the type of the outgoing ray.

6. Step 2: Stokes vector of the outgoing ray

6.1 Amplitude of electric field

For each of these 8 rays the amplitude of the electric field is its amplitude in the incident wave (the one which created this ray!) times corresponding "Fresnel coefficient". There are 8 ones from the table:

incident / outgoing	**Reflected**		**Refracted**	
Ordinary	ordinary	Extraordinary	ordinary	extraordinary
Extraordinary	ordinary	Extraordinary	ordinary	extraordinary

i.e. 8 complex-valued numbers:

$$R_{o\mapsto o}, R_{o\mapsto e}, R_{e\mapsto o}, R_{e\mapsto e}; \qquad T_{o\mapsto o}, T_{o\mapsto e}, T_{e\mapsto o}, T_{e\mapsto e}$$

where R is for reflection, T is for transmission and subscript explains the kinds of incident and outgoing rays. For example, $R_{e|\to o}$ is the ratio of electric field in the reflected ordinary ray to that in the incident extraordinary ray. Their calculation is a routine procedure [5, 6].

6.2 Stokes vectors

If the outgoing ray s (one among the 8) in a crystal is well distinct from the other, it has a "pure", linear polarization dictated by its type. If its direction is "indistinguishable" from that of another ray(s), they "merge" and the result has mixed polarization state.
Obviously, for either mixed or pure ray, the component $E_{||}$, parallel to the Stokes ort, comes (only) from the ordinary (outgoing) waves that relate to this ray, while $E_\perp$ comes (only) from the extraordinary (outgoing) waves that relate to this ray. Because of superposition, electric field amplitude for the outgoing ray s is

$$\begin{aligned} E_{||}^{(\text{out})} &= \delta_{s_{o\mapsto o},s} F_{o\mapsto o} E_{||}^{(\text{incid})} + \delta_{s_{e\mapsto o},s} F_{e\mapsto o} E_{\perp}^{(\text{incid})} \\ E_{\perp}^{(\text{out})} &= \delta_{s_{o\mapsto e},s} F_{o\mapsto e} E_{||}^{(\text{incid})} + \delta_{s_{e\mapsto e},s} F_{e\mapsto e} E_{\perp}^{(\text{incid})} \end{aligned} \tag{1}$$

where F is the Fourier (amplitude) coefficient (R or T depending on whether our outgoing ray is reflection on transmission) and

$$\delta_{u,v} \equiv \begin{cases} 1, & \text{if } u \approx v \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

so that summation goes over those outgoing rays which have direction s (or are very close to it). We only need the product of F and δ; this is the function of the incident ray direction and s:

$$\Phi_{o\mapsto e} \equiv \delta_{s_{o\mapsto e},s} F_{o\mapsto e} \tag{3}$$

The threshold of this "approximate equivalence" (2) is dictated by the particular problem. But since in an optical system of a finite size L the "most parallel" has angular spread $O(\lambda\ /\ L)$, normally we do not distinguish directions within at least that range.
The Stokes vector of this outgoing ray is by definition

$$\begin{aligned} I^{(\text{out})} &= \langle |E_{||}^{(\text{out})}|^2 \rangle + \langle |E_{\perp}^{(\text{out})}|^2 \rangle \\ Q^{(\text{out})} &= \langle |E_{||}^{(\text{out})}|^2 \rangle - \langle |E_{\perp}^{(\text{out})}|^2 \rangle \\ U^{(\text{out})} &= 2\,\mathrm{Re}\langle E_{\perp}^{(\text{out})} (E_{||}^{(\text{out})})^* \rangle \\ V^{(\text{out})} &= 2\,\mathrm{Im}\langle E_{\perp}^{(\text{out})} (E_{||}^{(\text{out})})^* \rangle \end{aligned} \tag{4}$$

Substituting the electric field components from (1) we calculate the quadratic averages, e.g.

$$\begin{aligned} \langle E_{\perp}^{(\text{out})} (E_{||}^{(\text{out})})^* \rangle = \Phi_{o\mapsto o}^* \Phi_{o\mapsto e} \langle |E_{||}^{(\text{incid})}|^2 \rangle \\ + \Phi_{e\mapsto o}^* \Phi_{e\mapsto e} \langle |E_{\perp}^{(\text{incid})}|^2 \rangle \\ + \Phi_{e\mapsto o}^* \Phi_{o\mapsto e} \langle E_{||}^{(\text{incid})} (E_{\perp}^{(\text{incid})})^* \rangle \\ + \Phi_{e\mapsto e} \Phi_{o\mapsto o}^* \langle (E_{||}^{(\text{incid})})^* E_{\perp}^{(\text{incid})} \rangle \end{aligned} \tag{5}$$

(the rest are calculated similarly and are dropped to save space). The quadratic averages of the incident field can be derived back from its Stokes vector, e.g.

$$\langle E_{||}^{(\text{incid})} (E_{\perp}^{(\text{incid})})^* \rangle = (U^{(\text{incid})} - iV^{(\text{incid})})/2$$

so composing the outgoing Stokes parameters from (5) gives

$$
\begin{aligned}
I^{(\text{out})} &= \frac{|\Phi_{o\mapsto o}|^2+|\Phi_{o\mapsto e}|^2+|\Phi_{e\mapsto o}|^2+|\Phi_{e\mapsto e}|^2}{2} I^{(\text{incid})} \\
&+ \frac{|\Phi_{o\mapsto o}|^2+|\Phi_{o\mapsto e}|^2-|\Phi_{e\mapsto o}|^2-|\Phi_{e\mapsto e}|^2}{2} Q^{(\text{incid})} \\
&+ \mathrm{Re}(\Phi_{o\mapsto o}\Phi^*_{e\mapsto o}+\Phi_{o\mapsto e}\Phi^*_{e\mapsto e}) U^{(\text{incid})} \\
&+ \mathrm{Im}(\Phi_{o\mapsto o}\Phi^*_{e\mapsto o}+\Phi_{o\mapsto e}\Phi^*_{e\mapsto e}) V^{(\text{incid})} \\
Q^{(\text{out})} &= \frac{|\Phi_{o\mapsto o}|^2-|\Phi_{o\mapsto e}|^2+|\Phi_{e\mapsto o}|^2-|\Phi_{e\mapsto e}|^2}{2} I^{(\text{incid})} \\
&+ \frac{|\Phi_{o\mapsto o}|^2-|\Phi_{o\mapsto e}|^2-|\Phi_{e\mapsto o}|^2+|\Phi_{e\mapsto e}|^2}{2} Q^{(\text{incid})} \\
&+ \mathrm{Re}(\Phi_{o\mapsto o}\Phi^*_{e\mapsto o}-\Phi_{o\mapsto e}\Phi^*_{e\mapsto e}) U^{(\text{incid})} \\
&+ \mathrm{Im}(\Phi_{o\mapsto o}\Phi^*_{e\mapsto o}-\Phi_{o\mapsto e}\Phi^*_{e\mapsto e}) V^{(\text{incid})} \\
U^{(\text{out})} &= \mathrm{Re}(\Phi^*_{o\mapsto o}\Phi_{o\mapsto e}+\Phi^*_{e\mapsto o}\Phi_{e\mapsto e}) I^{(\text{incid})} \\
&+ \mathrm{Re}(\Phi^*_{o\mapsto o}\Phi_{o\mapsto e}-\Phi^*_{e\mapsto o}\Phi_{e\mapsto e}) Q^{(\text{incid})} \\
&+ \mathrm{Re}(\Phi^*_{e\mapsto o}\Phi_{o\mapsto e}+\Phi_{e\mapsto e}\Phi^*_{o\mapsto o}) U^{(\text{incid})} \\
&+ \mathrm{Im}(\Phi^*_{e\mapsto o}\Phi_{o\mapsto e}-\Phi_{e\mapsto e}\Phi^*_{o\mapsto o}) V^{(\text{incid})} \\
V^{(\text{out})} &= \mathrm{Im}(\Phi^*_{o\mapsto o}\Phi_{o\mapsto e}+\Phi^*_{e\mapsto o}\Phi_{e\mapsto e}) I^{(\text{incid})} \\
&+ \mathrm{Im}(\Phi^*_{o\mapsto o}\Phi_{o\mapsto e}-\Phi^*_{e\mapsto o}\Phi_{e\mapsto e}) Q^{(\text{incid})} \\
&+ \mathrm{Im}(\Phi_{e\mapsto e}\Phi^*_{o\mapsto o}+\Phi^*_{e\mapsto o}\Phi_{o\mapsto e}) U^{(\text{incid})} \\
&+ \mathrm{Re}(\Phi_{e\mapsto e}\Phi^*_{o\mapsto o}-\Phi^*_{e\mapsto o}\Phi_{o\mapsto e}) V^{(\text{incid})}
\end{aligned}
\tag{6}
$$

Notice that by definition (2) the terms $\delta_{s_{o|\to e},s}\delta_{s_{o|\to o},s}$ and $\delta_{s_{e|\to e},s}\delta_{s_{e|\to o},s}$ entering these equations are 1 only when the "pure" (ordinary or extraordinary) incident ray produces coinciding ordinary and extraordinary outgoing rays. For crystal this is exceptional and requires special orientation of the normal, ray and optical axis. But if the outgoing ray is refraction into an isotropic medium, then this situation is common, because the ordinary and extraordinary rays in that medium always coincide, differing only in polarization direction.

7. Simulation example. Pled with birefringent tubes

7.1 Description of the device and simulation parameters

The device is a plane light emitter for an LCD display (its scheme is shown in Fig. 1). It consists of an light guiding plate (LGP) with reflectors attached to its bottom and side faces and light sources (array of LEDs) attached to one of the sides. Light enters the LGP at angles very tangent to its top and bottom faces, so in absence of scattering it would propagate the LGP without leaving it. But the LGP is glued to the top layer which is not homogeneous: sealed inside its "glass" are long anisotropic cylinders whose ordinary refraction coincides with that of "glass" while extraordinary differs from it. As a result, they scatter only extraordinary waves. This scattering makes the light to leave.

Naturally the fraction that leaves is determined by the concentration of anisotropic cylinders and there exists such a distribution of concentration for results in the best fit of desired luminance or illuminance. Our aim was to make spatial distribution of emitted light (over LGP) as achieve as uniform as possibly through manipulations concentration of cylinders. Each step of optimization is calculation of the output light distribution for given concentration profile and then updating the latter towards optimum [10]–[12].

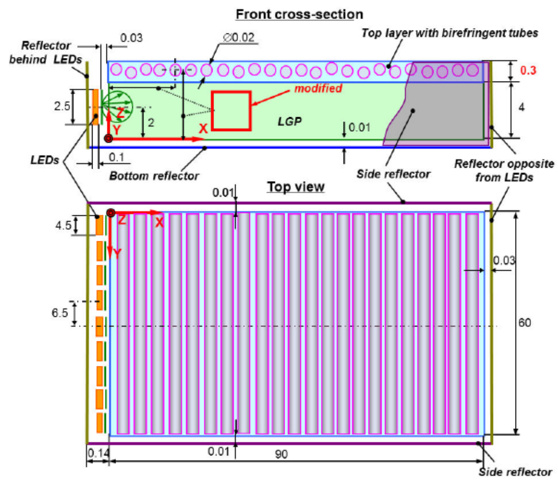

Рис. 1: Scheme of simulation model. Size units are mm; drawing is presented in artificial scales for better visualization. The "top layer" is glued (is in optical contact) with LGP.

The optical properties of the device parts are:
1. All LGP faces are Fresnel boundaries. LGP binder is clear medium with refraction index 1.49.
2. Reflectors opposite to the LED array and that behind the LEDs are Lambert with albedo 0.96.
3. Top layer and tubes are clear media have pure Fresnel properties. The tubes are birefringent with optical axis along the tube; refraction indices are η_e = 1.88 and η_o = 1.558. The binder is isotropic with refraction equal to the ordinary of the tubes, i.e. 1.558.
4. Bottom and side reflectors are mirrors with specular reflection 0.9.
5. LED box is Lambert with albedo 0.5

The LEDs are simulated as boxes with rectangular emitting area (4.5mm x 2.5mm along Y and Z axes correspondently). The angular distribution of the LED emission is Lambert. Each LED emits luminous flux is 11.11 lumens (so the 9 LEDs emit 100 lumens).

7.2 Results

Light emanated from the device is nearly 100% linearly polarized. This is because only extraordinary mode is scattered and thus leaves the device while the ordinary one propagates cylinders without any scattering and thus remains in the LGP.

As it is shown in Fig. 2 The spatial distribution of illuminance of the output surface for initial distribution of anisotropic tubes was highly non-uniform; after several optimization steps the density of tubes was found (Fig. 3) that made it nearly uniform.

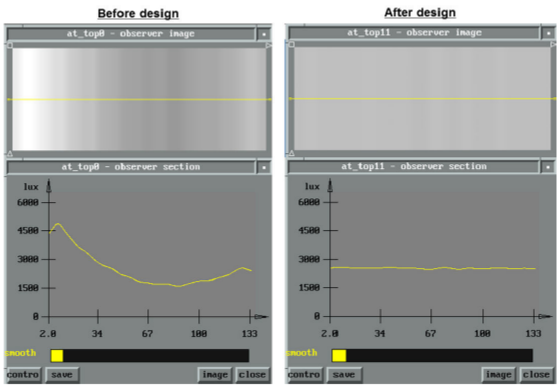

Рис. 2: Initial illuminance distribution (left) and final one (right)

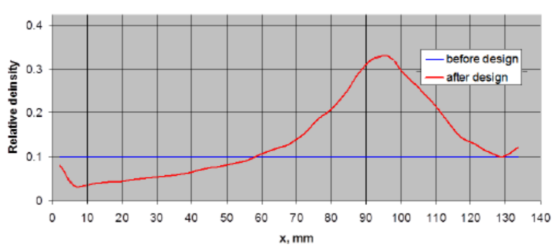

Рис. 3: Optimal concentration of tubes as a function of x

8. Appendix XYZ. Direction of scattered rays

Although the input and output data are ray parameters, intermediate calculations use wave vectors. First we calculate the wave vector of the incident ray; then calculate wave vectors of the outgoing waves, then convert them to the directions of the outgoing rays.

8.1 Ray to wave

Generally the ray s may contain both ordinary and extraordinary components (though usually only one). Notice s is a unit vector.
Direction of the ordinary wave coincides with the ray direction

$$\boldsymbol{k}_o = \boldsymbol{s}k_o \tag{7}$$

Direction of the extraordinary wave is calculated from the fact that because the wave vector k, optical axis a and ray direction s are in the same plane and the angle α between ray direction and optical axis is

$$\tan\alpha_e = \rho^2 \tan\beta_e$$

(see Section 98 of [2], eq. (98.6)). So, introducing the vector in the plane spanned by s and a orthogonal to a

$$\boldsymbol{b} \equiv \frac{\boldsymbol{s}-(\boldsymbol{a}\cdot\boldsymbol{s})\boldsymbol{a}}{|\boldsymbol{s}-(\boldsymbol{a}\cdot\boldsymbol{s})\boldsymbol{a}|} = \frac{\boldsymbol{s}-(\boldsymbol{a}\cdot\boldsymbol{s})\boldsymbol{a}}{\sin\alpha_e}$$

we have

$$\boldsymbol{k}_e = const\times(\boldsymbol{a}+\boldsymbol{b}\tan\beta_e) = const''\times((\rho^2-1)(\boldsymbol{a}\cdot\boldsymbol{s})\boldsymbol{a}+\boldsymbol{s}) \tag{8}$$

The scale factor is found from the condition on wave vector length

$$(1-\rho^2)(\boldsymbol{k}_e\cdot\boldsymbol{a})^2+\rho^2(\boldsymbol{k}_e\cdot\boldsymbol{k}_e)=k_o^2, \tag{9}$$

see ([2], eq. (98.2)). So,

$$\boldsymbol{k}_e = \frac{k_o}{\rho}\frac{\boldsymbol{s}+(\rho^2-1)(\boldsymbol{a}\cdot\boldsymbol{s})\boldsymbol{a}}{\sqrt{1+(\rho^2-1)(\boldsymbol{a}\cdot\boldsymbol{s})^2}} \tag{10}$$

which is another form of eq. (24) from [Debelov], but in their equation p is not unit vector and one must calculate its length, likely through field directions which is less convenient.

8.2 Direction of outgoing waves

8.2.1 Propagating wave

Generally there are 2 (ordinary and extraordinary) refracted waves and 2 (ordinary and extraordinary) reflected waves. Their wave vectors are in the plane of incidence ([1], Section 14.3.4, below eq. (23)), and their tangent component i.e. k_x is the same (Idid.). The y-component is 0 due to our choice of coordinate system.
For the ordinary outgoing wave:

$$\boldsymbol{k}_o = \left\{k_{x,\text{incid}},\, 0,\, \pm\sqrt{k_o^2-k_{x,\text{incid}}^2}\right\} \tag{11}$$

where "+" is for reflection and "–" is for refraction.
For extraordinary outgoing wave, again, the tangent component is $k_{e,x} = k_{x,incid}$ and $k_{e,y} = 0$. Then (9) is a quadratic equation in the unknown normal component $k_{e,z}$. It has two roots

$$k_{e,z} = k_{x,\text{incid}}\frac{(\rho^2-1)a_x a_z}{\rho^2(1-a_z^2)+a_z^2} \pm k_o\frac{\sqrt{\rho^2(1-a_z^2)+a_z^2-\frac{(a_x^2+a_z^2+a_y^2\rho^2)\rho^2}{k_o^2}k_{x,\text{incid}}^2}}{\rho^2(1-a_z^2)+a_z^2} \tag{12}$$

We must choose the one for which the ray direction is "away from the surface". The ray direction is given by (15), and after some algebra one obtains the sign of the normal component of ray direction:

$$\operatorname{sgn} s_{e,z} = \operatorname{sgn}\left(\pm k_o\sqrt{\rho^2(1-a_z^2)+a_z^2-\frac{(a_x^2+a_z^2+a_y^2\rho^2)\rho^2}{k_o^2}k_{x,\text{incid}}^2}\right)$$

where the + or − sign is the same as in (12). So, for reflection we choose "+" and for refraction "–" in (12):

$$k_{e,x} = k_{x,\text{incid}}$$
$$k_{e,y} = 0$$
$$k_{e,z} = \frac{(\rho^2-1)a_x a_z k_{x,\text{incid}}}{\rho^2+(1-\rho^2)a_z^2} \pm \frac{\sqrt{(1-\rho^2)\left(a_z^2 k_o^2 + k_{x,\text{incid}}^2\rho^2 a_y^2\right)+\rho^2(k_o^2-k_{x,\text{incid}}^2)}}{\rho^2+(1-\rho^2)a_z^2} \quad (13)$$

where "+" is for reflection and "–" is for refraction. Notice that when $k_{x,incid} > k_o\sqrt{\frac{\rho^2(1-\alpha_z^2)+\alpha_z^2}{(\alpha_x^2+\alpha_z^2+\alpha_y^2\rho^2)\rho^2}}$ the outgoing extraordinary wave becomes evanescent.

8.2.2 Evanescent wave

We use the same equations as for propagating wave, only now the wave vector is complex-valued. Scalar product actually just means

$$(\boldsymbol{u}\cdot\boldsymbol{v}) \equiv u_x v_x + u_y v_y + u_z v_z$$

(we do not apply complex conjugate to the 2^{nd} vector!). The rest is the same. Of the two roots we choose one for which k$_z$ corresponds to attenuating, not growing, wave.

8.3 Wave to ray

Eventually we calculate direction of rays and forget wave vectors. For ordinary waves, ray direction coincides with the wave vector:

$$\boldsymbol{s} = \boldsymbol{k}_o / |\boldsymbol{k}_o| \quad (14)$$

For extraordinary wave, $s = const \times (a + b tan\alpha_e)$ where b is orthogonal to a:

$$\boldsymbol{b} \equiv \frac{\boldsymbol{k}_e-(\boldsymbol{a}\cdot\boldsymbol{k}_e)\boldsymbol{a}}{|\boldsymbol{k}_e-(\boldsymbol{a}\cdot\boldsymbol{k}_e)\boldsymbol{a}|} = \frac{\boldsymbol{k}_e-(\boldsymbol{a}\cdot\boldsymbol{k}_e)\boldsymbol{a}}{\sqrt{1-(\boldsymbol{k}_e\cdot\boldsymbol{s})^2}} \equiv \frac{\boldsymbol{k}_e-(\boldsymbol{a}\cdot\boldsymbol{k}_e)\boldsymbol{a}}{\sin\beta_e}$$

After some trivial algebra this yields

$$\boldsymbol{s} = \frac{\boldsymbol{k}_e+(\rho^{-2}-1)(\boldsymbol{a}\cdot\boldsymbol{k}_e)\boldsymbol{a}}{\sqrt{k_e^2+(\rho^{-4}-1)(\boldsymbol{a}\cdot\boldsymbol{k}_e)^2}} \quad (15)$$

which is another form of eq. (23) from [6].

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