IMAGE DIFFUSION METHODS WITH INTEGRAL-BASED DIFFUSION COEFFICIENT

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Abstract

Diffusion filters are widely used to reduce noise on images. In [1] a new diffusion method for image filtering based on incorporating an integral of image intensity over a point neighborhood into the diffusion coefficient was suggested. Here we start by introducing two modified diffusion filters. Next we illustrate on number of images the advantages of modified filters comparing with not modified ones. In last section we introduce image sharpening method based on backward diffusion problem. We use constant diffusion coefficient and coefficient constructed using the incorporating an integral of image intensity over a point neighborhood. Benefits of using modified filter shown on the set of images.

Keywords: *image filtering, image sharpening, diffusion methods.*

1. INTRODUCTION

There are various applications of diffusion filtering [2-4]. One of the most popular applications is de-noising [5-7]. First attempts to add boundary preserving properties to diffusion were introduced in [8-9]. Authors used special diffusion coefficients, which were slowing down diffusion in high-gradient areas. This approach was developed in further works, for example [10-12] and [1].

During diffusion filtering image is smoothing. The noise level is reduced, but important features, like edges are also blurred. To slow down smoothing process on objects, preserving noise removal properties, we modify the diffusion coefficient. The target image class, which we use in our research, is images with text and high-contrast noise. In this case important objects are black letters on white background. White color we assume to have zero intensity value. We use an integral of image intensity over a point neighborhood as a 'noise detector'. The idea is that if the integral value of image intensity over a point neighborhood is high such point is most likely belongs to an object. If the integral value is low the point should be treated as a noise and should be blurred.

In this paper we show that using integral-based addition to diffusion coefficient in different diffusion methods could improve its properties on target image class.

2. DIFFUSION FILTERING USING INTEGRAL-BASED DIFFUSION COEFFICIENT

Let initial monochrome image $f(x,y)$ defined on rectangular domain $\Omega = \{(x,y), 0<x<a, 0<y<b\}$ with boundary Γ. Linear diffusion filter, Gaussian filter analog, with constant diffusion coefficient D is defined by the following problem

$$u_t = D^2 \Delta u,\ (x,y) \in \Omega,\ t>0, \tag{1}$$

$$\left.\frac{\partial u}{\partial n}\right|_{\Gamma} = 0,\ t>0, \tag{2}$$

$$u(x,y,0) = f(x,y),\ (x,y) \in \overline{\Omega}. \tag{3}$$

where n is normal to image boundary Γ. Filtered image $u(x,y,T)$ is the solution of this problem $u(x,y,t)$ taken at $t = T > 0$. Boundary condition (2) is used to preserve total amount of intensity on filtered images.

We modify diffusion coefficient by incorporating an integral of image intensity over a point neighborhood $O(x,y)$

$$D(x,y;u) = \left(1+\lambda^2 \int\limits_{O(x,y)} \overline{u}(s,p,t)\,ds\,dp\right)^{-1/2},\ \lambda>0,$$
$$\overline{u}(s,p,t) = \begin{cases} u(s,p,t) & (s,p)\in\overline{\Omega}, \\ 0 & (s,p)\notin\overline{\Omega}. \end{cases} \tag{4}$$

In this case diffusion filter will be defined as the following

$$u_t = div\left(\left(1+\lambda^2 \int\limits_{O(x,y)} \overline{u}(s,p,t)\,ds\,dp\right)^{-1} \nabla u\right),\ (x,y)\in\Omega,\ t>0, \tag{5}$$

$$\left.\frac{\partial u}{\partial n}\right|_{\Gamma} = 0,\ t>0, \tag{6}$$

$$u(x,y,0) = f(x,y),\ (x,y)\in\overline{\Omega}. \tag{7}$$

In the same we can modify diffusion filter defined by the following problem

$$u_t = div\left(D^2\left(|\nabla f|^2\right)\nabla u\right),\ (x,y)\in\Omega,\ t>0, \tag{8}$$

$$\left.\frac{\partial u}{\partial n}\right|_{\Gamma} = 0,\ t>0, \tag{9}$$

$$u(x,y,0) = f(x,y),\ (x,y)\in\overline{\Omega}. \tag{10}$$

where

$$D\left(|\nabla f|^2\right) = \left(1+\lambda^2|\nabla f|^2\right)^{-1/2},\ \lambda>0. \tag{11}$$

We modify it as the previous by incorporating an integral of image intensity over a point neighborhood

$$u_t = div\left(\left(1+\lambda^2|\nabla f|^2 \int\limits_{O(x,y)} \overline{u}(s,p,t)\,ds\,dp\right)^{-1} \nabla u\right),$$
$$(x,y)\in\Omega,\ t>0, \tag{12}$$

$$\left.\frac{\partial u}{\partial n}\right|_{\Gamma} = 0,\ t>0, \tag{13}$$

$$u(x,y,0) = f(x,y),\ (x,y)\in\overline{\Omega} \tag{14}$$

Since we used new diffusion coefficient the modified diffusion equations (5) and (12) become nonlinear.

3. COMPARING FILTERED IMAGES

To compare images filtered by initial and modified filters we use method suggested in [1]. Filtering time, the basic filter parameter is selected by the following algorithm:

• On original image we choose noisy area without useful objects. This area represented by the rectangle on the figures 1a and 2a.

• Next step is calculating integral of module gradient value from current solution over selected area G (shown on original images 1a and 2a as red rectangle): $V(t) = \int_G |\nabla u(x,t)| dx$. Calculated $V(t)$ we call 'noise volume'. It represents value of image intensity variation in calculation area. Since in this section we use positive diffusion coefficient the 'noise volume' function $V(t)$ is decreasing in time and tends to zero.

• Then we set desired value of 'noise volume', which we want to reach.

• On every time step we calculate $V(t)$ value. Diffusion process goes until we reach desired 'noise volume' in selected area of filtering image.

Calculations were performed based on the same numerical scheme as in [1]. Implicit numeric scheme is used. All the nonlinear parts are calculated on previous step by the time. The solution is found by the matrix sweep method. Local environment $O(x,y)$ is the square 3x3 pixels. Integral calculated using cubature formula on 9-point template.

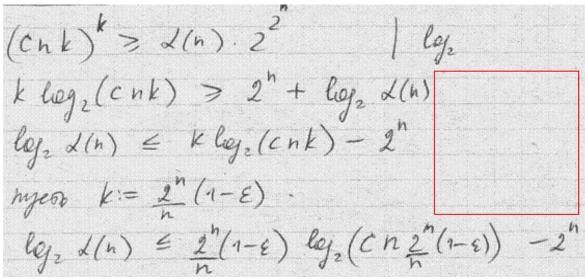


Fig. 1a. Original image

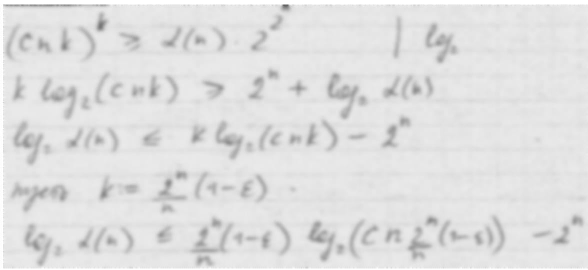


Fig. 1b. Image filtered using (1)-(3). 'noise volume' $V = 55$

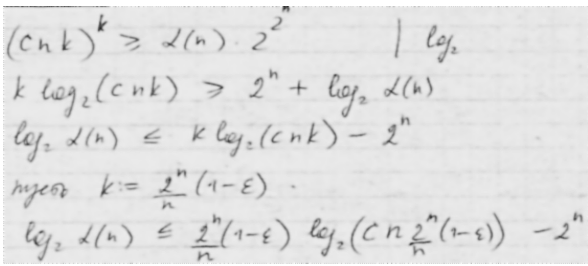


Fig. 1c. Image filtered using (5)-(7). $\lambda = 0.1$, 'noise volume' $V = 55$

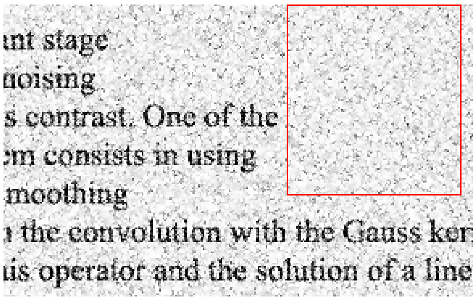


Fig. 2a. Original image

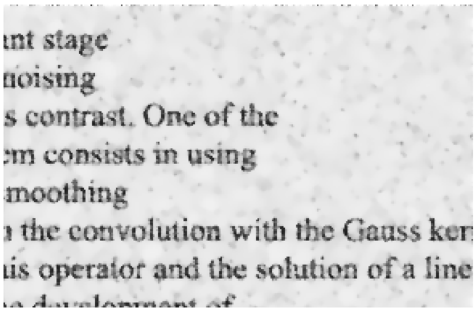


Fig. 2b. Image filtered using (8)-(10). $\lambda = 0.04$, 'noise volume' $V = 117$

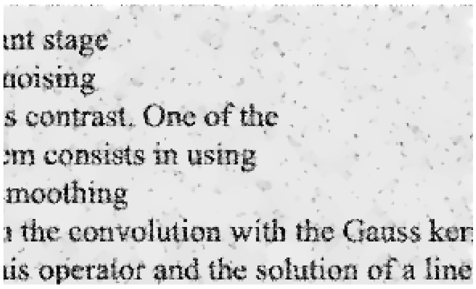


Fig. 2c. Image filtered using (12)-(14). $\lambda = 0.04$, 'noise volume' $V = 117$

Figures 1a and 2a represent original noisy images with specified area there we calculate 'noise volume'. On figures 1b, 1c, 2b and 2c we show filtered images. Image on figure 1b filtered using Gaussian filter and image on figure 1c using filter defined by the problem (5)-(7). Noise volume V in the selected area is the same for the both images. Image filtered by (8)-(10) is shown on figure 2b and filtering results of modified filter (12)-(14) are represented on figure 2a. As the same as in previous calculation, the 'noise volume' value is the same for both filtered images.

These results allow us to say that incorporating additional integral-based noise detector into diffusion coefficient could provide some advantages in comparison with original diffusion filtering methods.

4. IMAGE SHARPENING

There are many reasons why images are not sharp. It could be bad focusing or low photosensitivity of the image capturing device, lack scene illumination or problems in image reproduction cycle. Image processing like smoothing could also reduce image contrast. The goal of sharpening image is increasing contrast between useful objects and environment.

One of the sharpening methods is based on solving backwards diffusion problem:

$$u_t = D^2 \Delta u,\ (x,y) \in \Omega,\ t < T, \tag{15}$$

$$\left.\frac{\partial u}{\partial n}\right|_{\Gamma} = 0,\ t < T, \tag{16}$$

$$u(x,y,T) = \varphi(x,y),\ (x,y) \in \overline{\Omega}. \tag{17}$$

It is well known that noted problem (15)-(17) in general is ill posed. To solve this problem the quasiinversion method could be used [13].

Let original image $\varphi(x,y)$ defined on domain Ω with boundary Γ. Image $u(x,y,t_0)$, the solution of the following problem, taken at $t_0 > 0$ is contrasted analog of $\varphi(x,y)$

$$u_t = -D^2 \Delta u - \alpha \Delta\left(D^2 \Delta u\right),\ (x,y) \in \Omega,\ t > 0, \tag{18}$$

$$u|_{\Gamma} = 0, \quad \Delta u|_{\Gamma} = 0,\ t > 0, \Gamma = \partial\Omega, \tag{19}$$

$$u(x,y,0) = \varphi(x,y),\ (x,y) \in \overline{\Omega}. \tag{20}$$

During sharpening by the mentioned method contrast is increasing equally in all points. But useful objects have to be contrasted more than noise. Like in methods based on forward diffusion problem we include special ‘noise detector’ into sharpening method. We incorporate an integral of image intensity over a point neighborhood into the diffusion coefficient.

$$\sqrt{1 + \lambda^2 \int\limits_{O(x,y)} \overline{u}(s,p,t)\,ds\,dp} \tag{21}$$

Let define the nonlinear diffusion operator

$$A(u) = -div\left(\left(1 + \lambda^2 \int\limits_{O(x,y)} \overline{u}(s,p,t)\,ds\,dp\right)\nabla u\right) \tag{22}$$

So modified sharpening method is defined by the following problem:

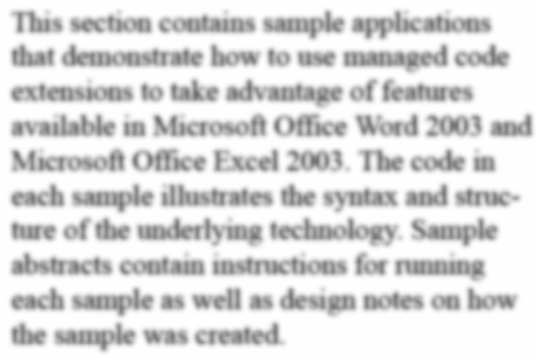
$$u_t = A(u) + \alpha A^2(u),\ (x,y) \in \Omega,\ t > 0, \tag{23}$$

$$u|_{\Gamma} = 0, \quad A(u)|_{\Gamma} = 0,\ t > 0, \Gamma = \partial\Omega, \tag{24}$$

$$u(x,y,0) = \varphi(x,y),\ (x,y) \in \overline{\Omega}. \tag{25}$$

5. COMPARING SHARPENING METHODS

To be able to correctly compare both methods we used special algorithm to calculate moment in time when we fix filtered images. We calculated maximum of gradient module on original image and on filtering image. We stop when maximum of gradient module value on current image became more then twice lager then initial one. By using this stop strategy we avoid large oscillations caused by ill-posedness of basic problem. We also calculated the value of regularization parameter α by straightforward enumeration in order to reduce oscillations. Following figures illustrates the advantages of modified filter.



This section contains sample applications that demonstrate how to use managed code extensions to take advantage of features available in Microsoft Office Word 2003 and Microsoft Office Excel 2003. The code in each sample illustrates the syntax and structure of the underlying technology. Sample abstracts contain instructions for running each sample as well as design notes on how the sample was created.

Fig. 3a. Original image

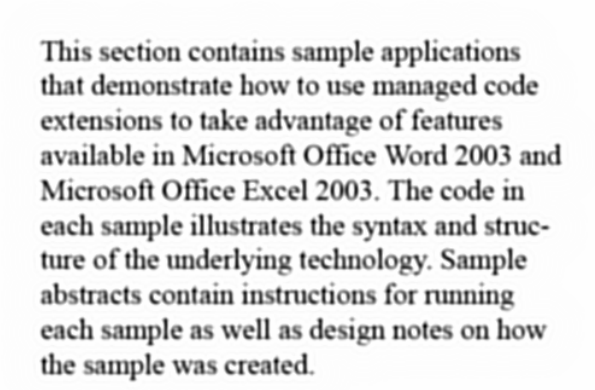


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Fig. 3b. Image contrasted using (18)-(20). $\lambda = 0.01$, $\alpha = 0.2$

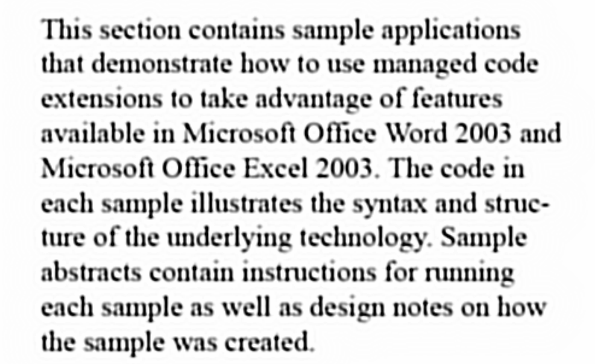

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Fig. 3c. Image contrasted using (23)-(25). $\lambda = 0.01$, $\alpha = 0.2$.

Figure 3a contains original blurred image. Image sharpening results are represented on figures 3b and 3c. Figure 3b shows picture processed using sharpening method based on the problem (18)-(20). Image sharpened by modified nonlinear diffusion filter (23)-(25) is represented on figure 3c.

6. CONCLUSION

In this work we show that useful properties could be added to different diffusion-based filtering and sharpening methods by including information about integral local intensity. We introduced new image sharpening method which is based on four-order partial differential equation and shown its advantages. In future work we focus on anisotropic diffusion filtering and including nonlinear source function into diffusion equation. This with combination of using integral-based noise detector should produce filtering and sharpening methods with new properties. Also we develop algorithms, which could be used to calculate filter parameters automatically.

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