EDGE DETECTION METHOD BY TIKHONOV REGULARIZATION

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Abstract

Image restoration is one of the classical inverse problems in image processing and computer vision, which consists in recovering information about the original image from incomplete or degraded data. This paper presents analytical solution for one-dimensional case of the Tikhonov regularization method and algorithm of parameter α selection by discrepancy, which finds the mostly smoothed function satisfying proximity constraints. Finally we present edge detection method based on Tikhonov regularization.

Keywords: regularization, image restoration, inverse problem.

1. INTRODUCTION

Reconstruction of an image from observed data is often an ill-posed inverse problem. Solution of such inverse problems can be achieved through regularization methods, which turn the problem into a well-posed one, and prevent the amplification of measurement noise during the reconstruction process. Thus, during the last few years nonlinear regularization [1] methods constitute an interesting alternative to such popular and well-founded tool as a diffusion filters. Therefore it is natural to investigate relations between both approaches. Scherzer and Weickert [2] emphasized that relations between diffusion and regularization theory exist via the Euler equation for the regularization functional. The regularization parameter and the diffusion time can be identified if one regards Euler equation for regularization as time-discrete diffusion with single implicit time step.

In this paper we present analytical solution for one-dimensional case of the Tikhonov regularization method and algorithm of parameter α selection by discrepancy, which finds the mostly smoothed function satisfying proximity constraints. Finally we present edge detection method based on Tikhonov regularization.

2. TIKHONOV REGULARIZATION METHOD

The method based on Tikhonov's regularization [3] looks as follows:

For a function unknown data $\overline{u} \in W_2^1$, we have an approximation $u_\delta : \|\overline{u} - u_\delta\|_{L_2} \le \delta$. We want to find $\widetilde{u} \in W_2^1$, such that $\|\tilde{u} - \overline{u}\|_{W_2^1} \to 0, \quad \delta \to 0$.

This problem is solved by minimizing the Tikhonov functional:

$$E_\alpha(\widetilde{u}) = \|\widetilde{u} - u_\delta\|_{L_2}^2 + \alpha \left\| \frac{d}{dx}\widetilde{u} \right\|_{L_2}^2$$

The first term in functional is a data fidelity term, while the second term rewards smoothness.

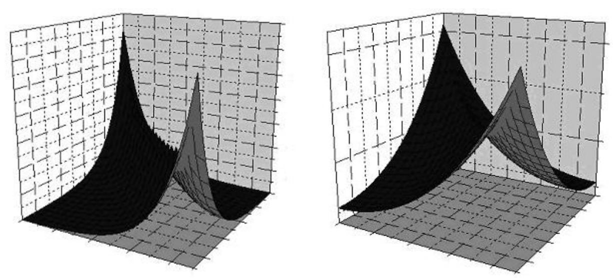
The problem of the Tikhonov functional minimization is reduced to solving the Euler equation for u_α

$$\begin{cases} -\alpha u_\alpha'' + u_\alpha = u_\delta \\ u_\alpha'(1) = u_\alpha'(-1) = 0 \end{cases}$$

Its solution $u_\alpha(x) = \int_{-1}^{1} G_\alpha(x,s) u_\delta(s) ds,$ with the kernel

$$G_\alpha(x,s) = \begin{cases} \dfrac{1}{\sqrt{\alpha}\, sh \dfrac{2}{\sqrt{\alpha}}} ch \dfrac{1}{\sqrt{\alpha}}(x-1) ch \dfrac{1}{\sqrt{\alpha}}(s+1) & s \le x \\ \dfrac{1}{\sqrt{\alpha}\, sh \dfrac{2}{\sqrt{\alpha}}} ch \dfrac{1}{\sqrt{\alpha}}(s-1) ch \dfrac{1}{\sqrt{\alpha}}(x+1) & s \ge x \end{cases}$$

The graphs of the kernel look as follows:



α=0.1 α=10

Fig. 1. Graphs of the kernel $G_\alpha(x,s)$,

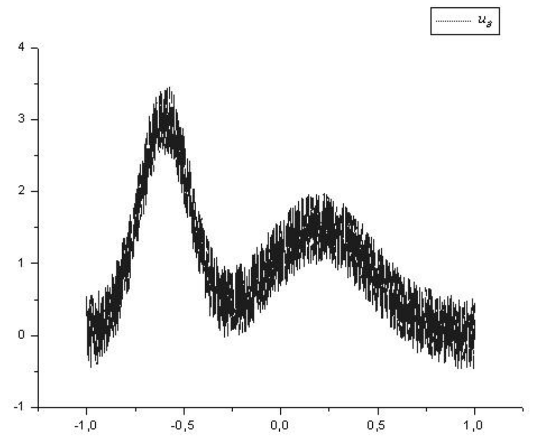
$-1 < x < 1, -1 < s < 1$.

2.1 Parameter α selection by discrepancy:

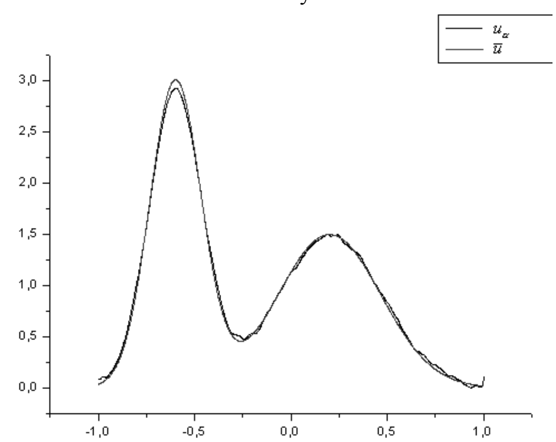
To find $\alpha = \alpha(\delta)$, the solution $u_\alpha(x)$ must satisfy the following equation [3,4]:

$$\varphi(\alpha) = \int_{-1}^{1} \left(u_\alpha(x) - u_\delta(x)\right)^2 dx = \delta^2$$

The proposed method finds function u_α with the minimum norm of the derivative in L_2 among all the functions u from the set $\|u - u_\delta\|_{L_2} \le \delta$.



Source noisy data



Function restoration

Fig. 2. Parameter α selection by discrepancy method (δ^2=0.165, α=0.000986).

2.2 Tikhonovs functional applications for edge detection.

A very interesting consequence of the above approach is the fact that the solution satisfies the equation $-\alpha u'' + u = u_\delta$. So, zero crossing of the second derivative is equivalent to the change of the sign of $u - u_\delta = 0$. This allows us to construct a method of stable second derivative zero crossing detection. Its application to image processing gives us a new method of stable edge detection.

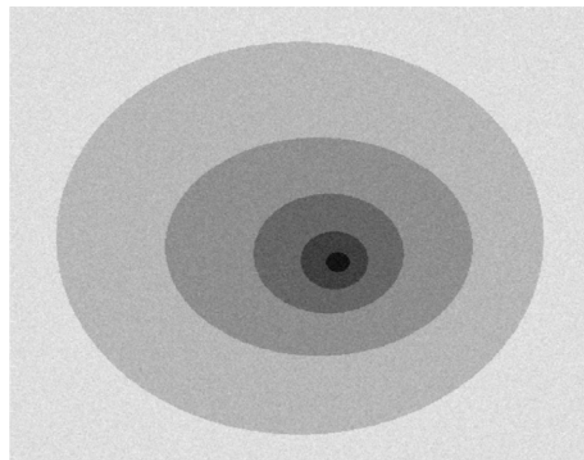
From equation $-\alpha u'' + u = u_\delta$

follows $u'' = 0 \Leftrightarrow u - u_\delta = 0$.

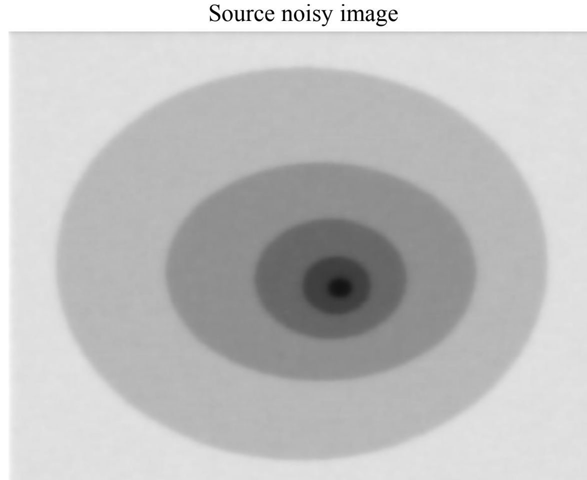
So we smooth every row of the noisy image u_δ, and find the second derivative in the y directions $\dfrac{\partial^2 u}{\partial y^2}$. Then we smooth every column of the initial image u_δ, and find the second derivative in the y directions $\dfrac{\partial^2 u}{\partial x^2}$. Summing these results, we find non-directional second derivative operator (the Laplacian operator: $\Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}$).

The proposed method enables us to obtain edges of an image basing on Laplacian zero crossing [5,6] analysis. At the same time our regularization method is very efficient as a preliminary smoothing algorithm for more advanced methods of edge detection like Canny [7] edge detector.

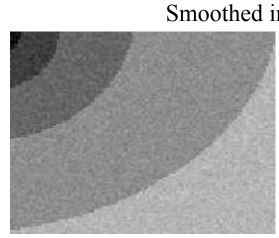
Figure 3 shows the applying of the following algorithm for edge detection.



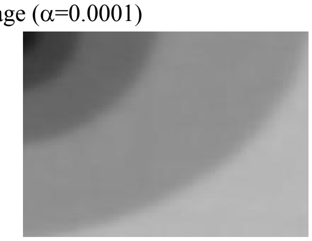
Source noisy image



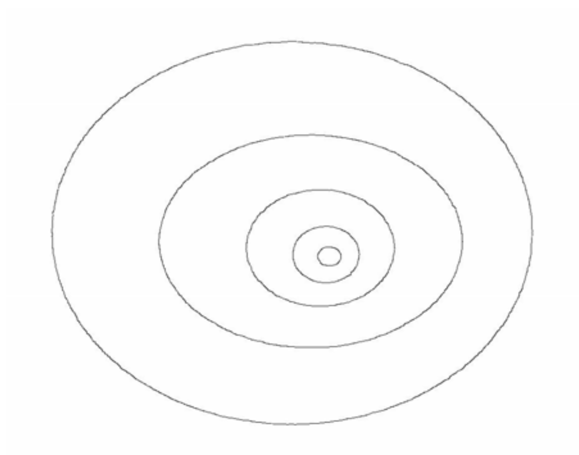
Smoothed image (α=0.0001)



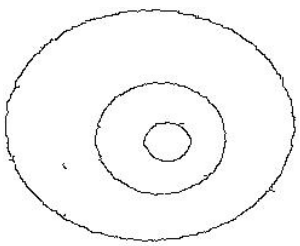
Detail of a source noisy image



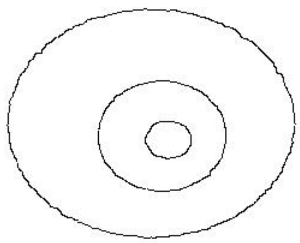
Detail of a smoothed image



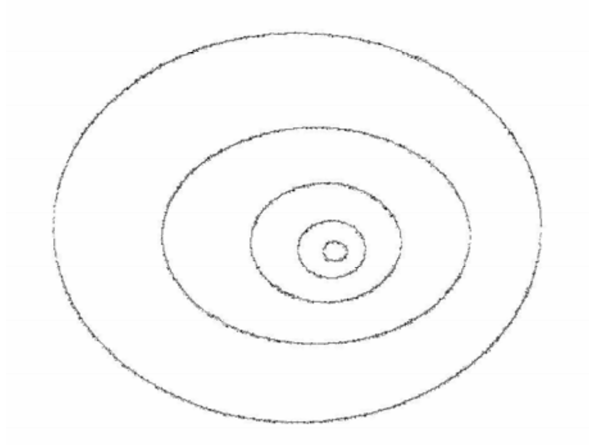
Canny edge detector result for smoothed image



Detail of Canny edge detector result for source image



Detail of Canny edge detector result for smoothed image



Laplassian zero crossing obtained by our method

Source image

Laplassian zero crossing obtained by our method

Fig. 3. Edge detection results.

The derivatives for gradient, used for edge thresholding, can be calculated by the formula for derivative

$$u'(x) = \frac{1}{\sqrt{\alpha}\, sh \frac{2}{\sqrt{\alpha}}} \int_{-1}^{x} sh \frac{1}{\sqrt{\alpha}}(x-1) ch \frac{1}{\sqrt{\alpha}}(s+1) u_{\delta}(s) ds -$$

$$- \frac{1}{\sqrt{\alpha}\, sh \frac{2}{\sqrt{\alpha}}} \int_{x}^{1} ch \frac{1}{\sqrt{\alpha}}(s-1) sh \frac{1}{\sqrt{\alpha}}(x+1) u_{\delta}(s) ds.$$

3. CONCLUSION

This paper presents analytical solution for one-dimensional case of a Tikhonov regularization method used for image edge detection. An obtained result allows to make conjecture about behavior of more complicated nonlinear regularization methods, evaluate their influence on the edges and more precisely select their parameters.

ACKNOWLEDGEMENTS

This research was supported by AFOSR grant CRDF # RP0-1391-MO-03.

4. REFERENCES

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