

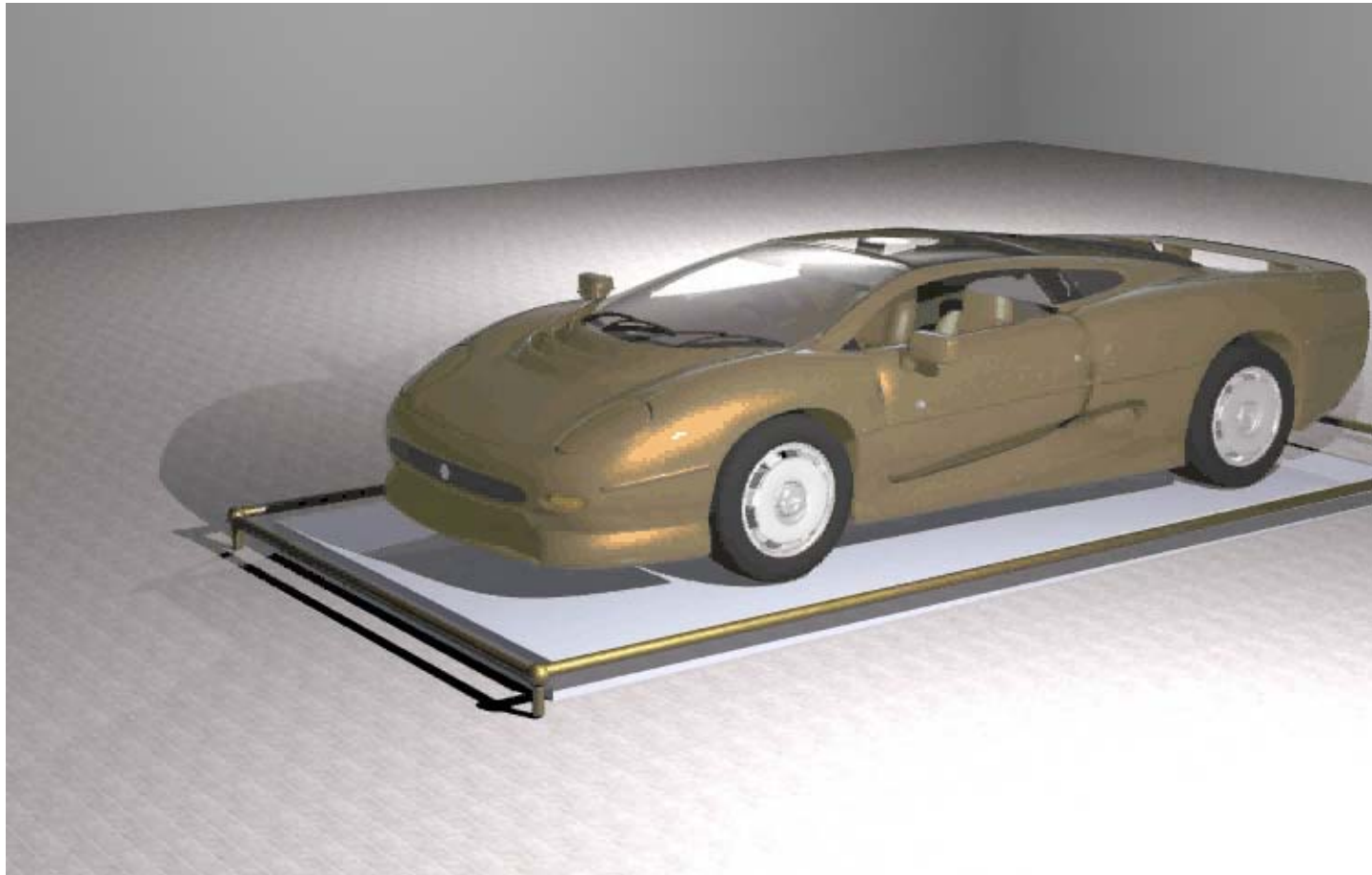
# Simulation of sparkles of metallic paints

S.V.Ershov, K.V.Kolchin, A.B.Khodulev

*Keldysh Institute for Applied Mathematics*

Sparkles are tiny bright point (flashes) on paint surface. They are pretty visible at close distance in a sunny day; but fade with distance as well as under diffuse illumination.

*Zoom in:*



International Conference Graphicon 1999,  
Moscow, Russia, <http://www.graphicon.ru/>

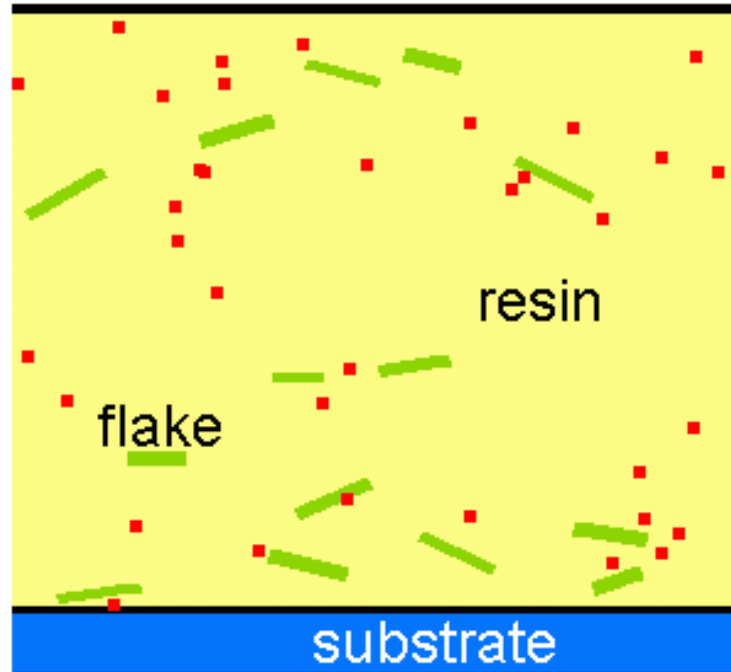
*Illumination: clear sky -> cloudy sky:*



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# 1. Paint structure

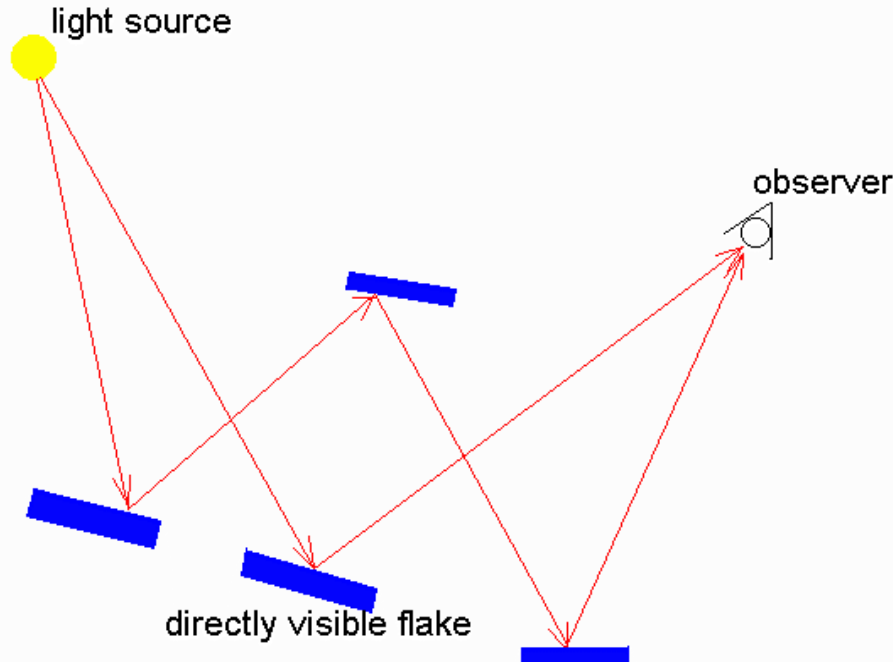
Metallic paints are made of *substrate* (color base) above which laid is a layer of resin with *flakes* and *pigments* dispersed in it.



- **Substrate** is a layer of solid (diffuse) paint
- **Flakes** are small platelets, optionally with interference coating
- **Pigment** particles are very tiny (Mie or Rayleigh) particles

## 2. Sparkles Are Flakes

Flakes are *flat smooth platelets*, acting as tiny mirrors. So reflection by an individual flake is purely *specular*. If a ray from a point or parallel LS is reflected by this flake into an eye, we see a very bright point:



So for some flakes (with “fortunate” orientation) a ray from light source undergoes single reflection. For other flakes (whose orientation is different) there are many reflections. Each reflection attenuates light, so these flakes are less bright.

Also, flakes can be illuminated by diffuse light scattered by pigments... this is also less bright light.

# 3. Basic Assumptions

- We assume that paint contains single layer with flakes of the same structure (if there are several kinds of flakes, we need to treat them separately, then add luminances)
- We assume that only directly visible flakes contribute to sparkling. Other flakes as well as pigments etc. only contribute to “regular” (not fluctuating) component of luminance. Indeed, conditions of reflection are random so if a ray undergoes many independent reflections, the resulting attenuation is averaged with fluctuations being weaker than for a single reflection.
- We assume that sparkles are only due to illumination by point or parallel light sources. Observer has finite angular size  $\ll$  light source angular size.
- We assume that size, orientation and depth of different flakes are statistically independent. Each of the above characteristics is a random variable whose distribution is assumed to be known. Namely, size and orientation are normal deviates, and depth is a uniform deviate.



# 4. Algorithm

- For each pixel, determine how many sparkles in it.  
 $\langle \# \text{ sparkles} \rangle = \langle \# \text{ flakes} \rangle \times \Pr(\text{flakes has fortunate orientation})$   
 $\langle \# \text{ flakes} \rangle = \text{surface density of flakes} \times \text{area of pixel projection}$   
The above probability is the distribution of flake normal times the solid angle of deviation of flake normal for which it still reflects light into eyes.  
The above deviation is because light source (e.g. sun) has finite size, so flake normal may deviate a bit from bisector of directions to camera and light source.
- The actual count of sparkles is a Poisson deviate with above mean. So pick at random the actual count of sparkles in this pixel.
- For all  $n$  sparkles, take at random the size of each, and depth of each (attenuation of light  $\sim$  optical density thickness of paint medium it ran through)
- Calculate the luminance of each sparkle, then sum  $\Rightarrow$  sparkling of pixel.

- ? Add the above luminance to “usual” paint luminance?

This would have been a mistake! For very large density of flakes, fluctuations would be averaged, luminance of any pixel would be ~ the same and equal to the part of paint luminance created by flakes.

So we would have accounted for flake luminance twice!

- So a correct approach is to subtract *average flake luminance (for this pixel)*.

Then in case of large flake density we shall first subtract average flake luminance and then again add it. No change, as expected. For very small density, in pixels where there are now sparkles we shall just see only *other* components of paint luminance (substrate luminance etc).



# 5. Formulae

- **The probability that a flake is a sparkle:**

$$\text{Pr}(\text{specular reflection}) = \Delta^2 P(\beta) \frac{\cos \vartheta}{8\eta^2 \cos \alpha \sqrt{1 - \eta^{-2} \sin^2 \vartheta}}$$

Here  $\Delta$  is the angular size of light source (that of camera is much less),  $\vartheta$  is the angle of incidence,  $\beta$  is flake inclination (the angle between flake normal and paint normal),  $\alpha$  is the angle between incident ray and flake normal, and  $\eta$  is the index of refraction of resin.  $P()$  is the distribution of orientation.

- **Optical density of paint (attenuation of light) in paint**

$$\tau(\vartheta) = D \langle S \rangle \times r(\sqrt{1 - \eta^{-2} \sin^2 \vartheta})$$

Here  $D$  is density of flakes,  $\langle S \rangle$  is mean flake area and  $r$  is flake reflectance as a function of angle of incidence.  $\vartheta$  is the angle between ray and paint normal.

The above value is thickness for unit depth.

- **Luminance of flake**

$$L = I \frac{(1 - r_\eta(\vartheta))(1 - r_\eta(\vartheta'))}{\pi \Sigma \sqrt{1 - \eta^{-2} \sin^2 \vartheta'} \sqrt{1 - \eta^{-2} \sin^2 \vartheta}} \cos \alpha r(\alpha) S e^{-z[\tau(\vartheta') + \tau(\vartheta)]}$$

Here  $S$  is flake area and  $\Sigma$  is the area of pixel's projection onto paint surface,  $r_\eta$  is reflection of Fresnel boundary "paint-air".  $I$  is illumination of surface point.

- **Average luminance of flakes (= flake part of BRDF):**

$$\langle L \rangle = I \times \pi D \langle S \rangle e^{-\tau(\vartheta) - \tau(\vartheta')} r(\alpha) \frac{(1 - r_\eta(\vartheta))(1 - r_\eta(\vartheta'))}{4\eta^2 \cos \vartheta' \sqrt{1 - \eta^{-2} \sin^2 \vartheta}} P(\beta)$$

# 6. The Angles

